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# Reliability of Trusses under Corrosion

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# **Reliability of Trusses under Corrosion**

by

Lutong Zhang

A Thesis

Presented to the Graduate and Research Committee  
of Lehigh University  
in Candidacy for the Degree of  
Master of Science

in

Structural Engineering

Department of Civil and Environmental Engineering  
Lehigh University  
Bethlehem, Pennsylvania

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for the Master of Science.

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Date

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Thesis Advisor

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Chairperson of Department

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The opinions, findings, and conclusions presented in this study are those of the author and do not necessarily reflect the views of the individuals acknowledged above.

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## Abstract

The aim of this thesis is to develop a method for reliability analysis of truss structures that is appropriate for numerical calculation. A new method that quantifies truss structural reliability is established. Reliability of component is evaluated by computer simulation methods. The accuracy of Monte Carlo simulation (MCS) and the perturbation method have been used. However, since the MCS method provides more accurate results, it has been used to calculate component reliability.

With the information about component reliability, a methodology to calculate the time-variant system reliability is established based on Markov Chain theory. The long term behavior of a truss structural system can be modeled by a Markov chain process. Finally, a Matlab program has been written based on the methods mentioned above, and two case studies have been presented to evaluate the time-variant reliability of two different ten-bar truss structures.

## Chapter 1

### Introduction

Trusses have been widely used because of their simplicity and efficiency. A truss is usually composed of straight bar members whose ends are connected at joints with hinges referred to as nodes. External loads act only on nodes so that a bar component is either under conditions of compression or tension. By combining bar components in different patterns and orientations, it is easy to form different types of trusses.

Structural failure modes are identified by the associated limit state equations. In order to achieve the final goal to quantify reliability, a truss structure is regarded as a combination of subsystems which contain relatively small groups of bar components. The overall system reliability is calculated based on the reliabilities of each subsystem, which are governed by the corresponding limit state equations [1]. A significant feature of truss structures is that they have multiple interdependent failure modes.

The fast development of computational technology enables civil engineers to analyze large scale reliability problems by numerical methods [2]. Because of its simplicity, the matrix analysis method becomes one of the most fundamental numerical methods for structural analysis. The overall structure stiffness matrix is generated by superimposing the stiffness matrices of each component

$$[K][d] = [f] \quad (1.1)$$

where  $[K]$  is the global stiffness matrix,  $[f]$  is the vector of external loads, and  $[d]$  is the nodal displacement vector. This method not only provides displacements at each node and deformations in each bar component, but offers a criterion to identify the

failure conditions [3]. However, as mentioned by Murotsu [3], in case of the statically indeterminate trusses, there are too many failure modes. Hence, the system failure probability is estimated by calculating its lower and upper bounds. The limitation of this method is that it only provides an estimated interval for system reliability, which is bounded by the upper and lower bounds.

In this thesis, a new method that quantifies truss structure reliability is established. This new method is based on matrix analysis and the theory of Markov chain.

The working condition of a truss structure at a certain point in time is a function of a set of variables, such as the cross section areas ( $A_i(t)$ ) of each bar component. These variables span a continuous sample space. A continuous sample space can be discretized into a finite state space. That is to say, at a certain point in time, all the possible working conditions of a truss structure can be represented by a finite set of states, which are named as system states. Additionally, a truss structural system can move from one system state to another over a period of time  $T$ , and the associated transition probability can be evaluated by numerical simulation. Two states are connected by a transition probability, so a map connecting all the states can be generated. This kind of map indicates the long-term behavior of a truss structure over its lifetime.

There are three benefits of this method. Firstly, this method is suitable for the reliability analysis of a truss that has a large number of components. Secondly, this method is computational efficient. Finally, this method not only evaluates truss system reliability, but also provides detailed data which reveals the evolution of the truss structural system over its lifetime.

## 1.1 Objectives

The primary objective of this study is to develop a reliability analysis procedure for trusses. Bar components are the most fundamental structural units of a truss. Hence, a computational affordable simulation method is needed to quantify the reliabilities of each component. The possible working conditions of a truss structure at a certain point in time are defined as system states, and the system can move from one state to another. Knowing component reliabilities, a graph which describes probability transitions among system states is established based on the theory of Markov chain, and the system reliability is calculated based on this Markov process.

## 1.2 Research approach

In order to achieve the objectives presented in section 1.1, the research approach consists of the following steps:

1. Clarify basic concepts of uncertainties in engineering problems
2. Clarify basic concepts of component reliability and system reliability
3. Clarify basic concepts of Markov chain theory
4. Establish a computer simulation method to quantify the reliabilities of truss members
5. Test Monte Carlo simulation and perturbation method, and then decide which one is more applicable for calculation of component reliability.
6. Establish a procedure to model the working conditions of a truss structure and quantify the transition probabilities among the system states.
7. Calculate system reliability of the truss structure by the Markov chain procedure.
8. With all the methods established, write a computer program to perform reliability analysis for a truss structure.

### **1.3 Organization of this study**

This study is organized into seven chapters. Chapter 1 is a basic introduction about this study. Chapter 2 introduces basic concepts about the theory of probability, the theory of reliability, and the theory of Markov chain. Chapter 3 discusses the topics of component reliability. Chapter 4 establishes a method to calculate system reliability. Chapter 5 introduces a method to model corrosion of a truss structure. Finally, Chapter 6 presents two case studies, and Chapter 7 states the conclusions of this study.

## Chapter 2

### Basic concepts

#### 2.1 Randomness in engineering problems

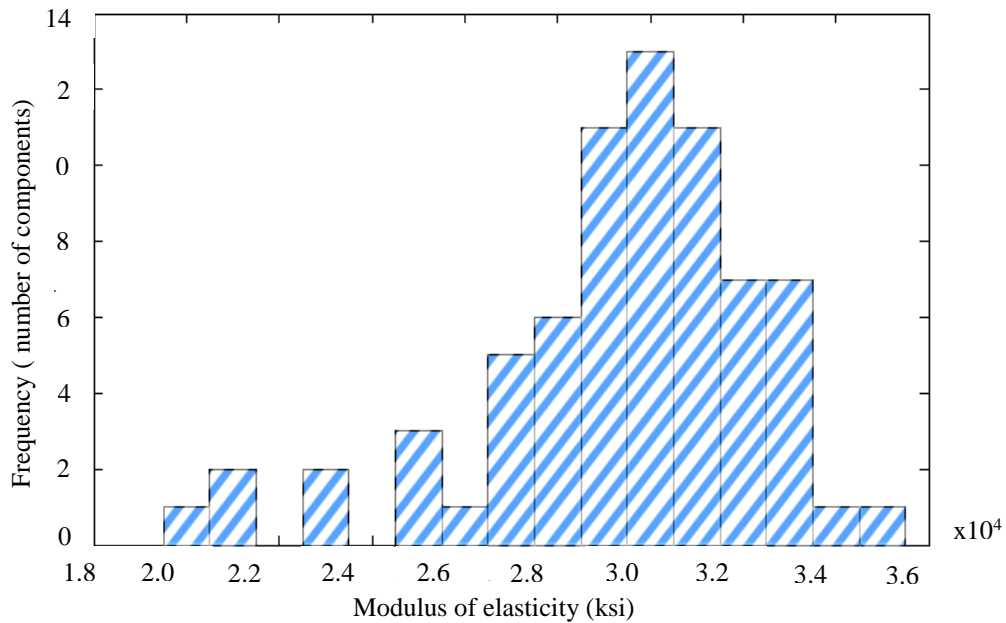
Uncertainties are unavoidable in real world problems. There are different sources of uncertainties. For example, steel is a mixture of iron and carbon, so it has different values of Young's modulus at different points. Secondly, steel components are fabricated by a set of complex processes such as bending, cutting or rolling. Uncertainties tend to accumulate after each process. Thirdly, the external environment of a structure is unpredictable. Environmental loads, such as the wind load and the snow load may vary with time randomly, and most of the natural disasters are unpredictable as well. The long period deteriorations, such as corrosion, are also unpredictable.

For these reasons, the long term behavior of a truss structure is uncertain. Therefore, the random behavior of a truss structure can be described by the theory of probability.

#### 2.2 Basic concepts of the probability theory

According to the probability theory, there are basically two different types of uncertainties [4]. The first type of uncertainty is identified as aleatory uncertainty. For example, Hess and Ayyub tested the Young's modulus of a certain kind of steel component in 1997 and their results are presented in Figure 1 [5].

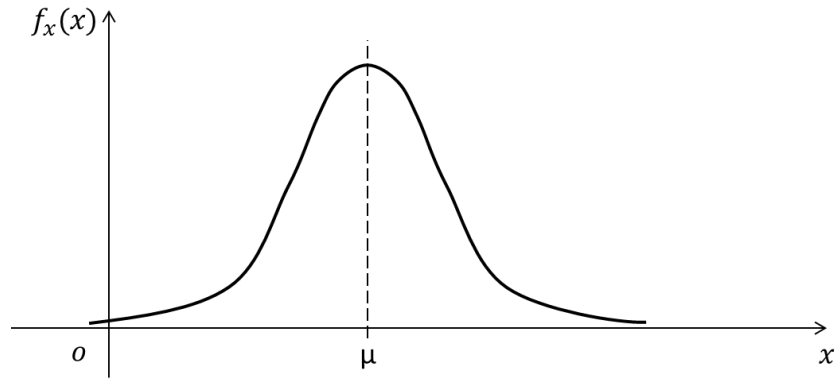




**Figure 1 Test Results of Modulus of Elasticity by Hess and Ayyub (1997) [5]**

This figure is a histogram of the Young's modulus of the test components. It indicates the distribution of the Young's modulus for the tested bar components. Generally speaking, for a certain type of component, this kind of histograms tends to follow a certain type of distribution. Another kind of uncertainty is identified as the epistemic uncertainty, which is caused by the imperfect knowledge [4].

Probability is defined as follows [9]: Given a measurable space  $(\Omega, A)$ , a function  $P: A \rightarrow R$  is called a probability measure and  $(\Omega, A, P)$  is called a probability space if  $P$  satisfies the Kolmogorov axioms. Here the  $\Omega$  is a sample space. The uncertain properties of a component such as the cross section area, the ultimate stress and the Young's modulus can be described as random variables satisfying certain types of distribution. Figure 2 shows a probability density function (PDF) of a random variable.



**Figure 2 PDF of a random variable**

In Figure 2, on the horizontal axis,  $\mu$  stands for the mean value of this random variable. The dispersion of this random variable is commonly described by the standard deviation  $\sigma$ .

### **2.3 Basic concepts of reliability analysis**

Traditionally, the fundamental design goal is to achieve a structural capacity that meets the worst demand condition [4]. However, both the demand and the capacity are functions of random variables such as cross section areas and material properties, so they are random variables themselves. Hence, the reliability analysis is employed.

Reliability is defined as the probability that an item will adequately perform its specified purpose for a specified period of time under specified environmental conditions [4]. The word “item” in this definition means the object to study [4]. When analyzing the reliability of a component, the “item” stands for each bar component. On the other hand, when analyzing the reliability of the system, “item” here represents the whole truss structure. Probability theory is the foundation of reliability analysis.

Here we define the following random variables:

$$X = \text{Capacity of a structure} \quad (2.1)$$

$$Y = \text{Demand on a structure} \quad (2.2)$$

Thus the probability of failure [4] is

$$p_F = P(X < Y) \quad (2.3)$$

and the associated probability of safety can be defined as [4]:

$$p_S = 1 - p_F \quad (2.4)$$

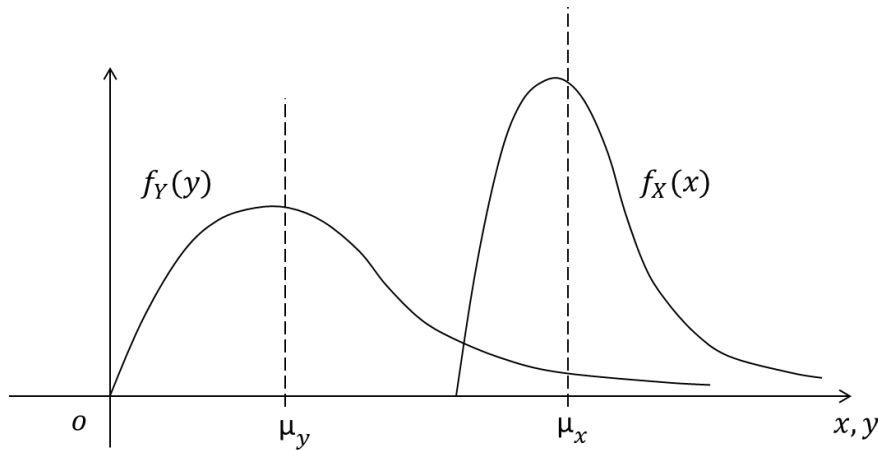
For discrete system [4],

$$p_F = \sum_{\text{each } y} P(Y = y) P(X < Y | Y = y) \quad (2.5)$$

And for continuous system [4],

$$p_F = \int_0^{\infty} F_x(y) f_Y(y) dy \quad (2.6)$$

$F_x$  and  $F_y$  are the probability distribution functions of random variable X and Y.  $f_x$  and  $f_y$  are PDFs for random variable X and Y.



**Figure 3 PDF functions of capacity(X) and demand(Y)**

In Figure 3, there are 2 PDF curves. One of them is the demand PDF curve ( $f_Y$ ) and the other one is the PDF for the capacity ( $f_X$ ). If these two curves move toward each

other, the overlap area will increase and the system reliability will decrease. The relative position of  $f_X$  and  $f_Y$  can be described by [4]:

$$\mu_X/\mu_Y \quad (2.7)$$

which is named as the “central safety factor” [4]. Because the shape of the PDF curve will also influence the system reliability, the central safety factor is not enough to quantify the system reliability.

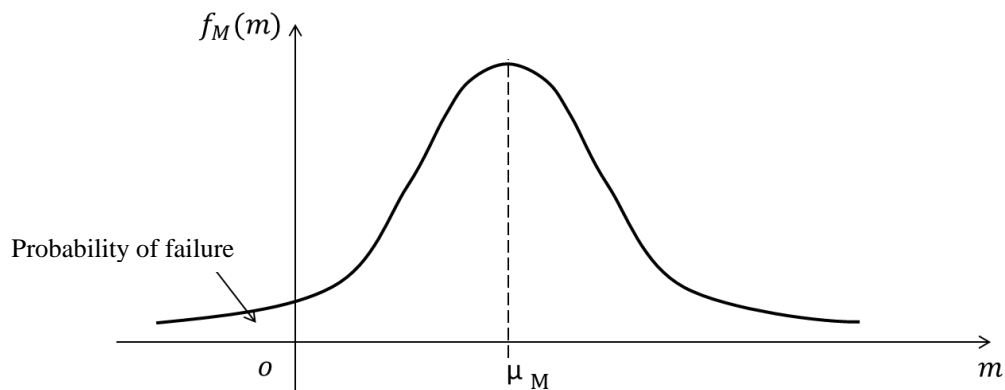
Suppose  $X$  and  $Y$  are statistically independent random variables and their joint PDF is defined as  $f_{XY}(x, y)$ . A better way to look at this capacity-demand problem is to define a new random variable [4]:

$$M = X - Y \quad (2.8)$$

Generally,  $M$  is named as “safety margin” [4]. PDF of  $M$  is  $f_M(m)$ . The probability of failure is [4]:

$$p_F = \int_{-\infty}^0 f_M(m) dm \quad (2.9)$$

The PDF of  $M$  is shown in Figure 4, and the area under PDF curve on the left side of the vertical axis equals the probability of failure.



**Figure 4 PDF of margin of safety**

Assume X and Y are normally distributed. Their mean values and standard deviations are  $\mu_X$  and  $\sigma_X$  for X, and  $\mu_Y$ ,  $\sigma_Y$  for Y. And then:

$$p_S = \Phi\left(\frac{\mu_M}{\sigma_M}\right) \quad (2.10)$$

Because:

$$\mu_M = \mu_X - \mu_Y \quad (2.11)$$

And:

$$\sigma_M^2 = \sigma_X^2 + \sigma_Y^2 \quad (2.12)$$

We have:

$$p_S = \Phi\left(\frac{\mu_X - \mu_Y}{\sqrt{\sigma_X^2 + \sigma_Y^2}}\right) \quad (2.13)$$

Finally, the ratio  $\frac{\mu_M}{\sigma_M}$  is defined as the reliability index, and it is expressed as [4]:

$$\beta = \frac{\mu_M}{\sigma_M} = \frac{\mu_X - \mu_Y}{\sqrt{\sigma_X^2 + \sigma_Y^2}} \quad (2.14)$$

In reality, random variables may not be normally distributed. So the reduced variables are defined as [4]:

$$X' = \frac{X - \mu_X}{\sigma_X} \quad (2.15)$$

$$Y' = \frac{Y - \mu_Y}{\sigma_Y} \quad (2.16)$$

So,

$$X = \sigma_X X' + \mu_X \quad (2.17)$$

$$Y = \sigma_Y Y' + \mu_Y \quad (2.18)$$

Recall:

$$M = X - Y \quad (2.19)$$

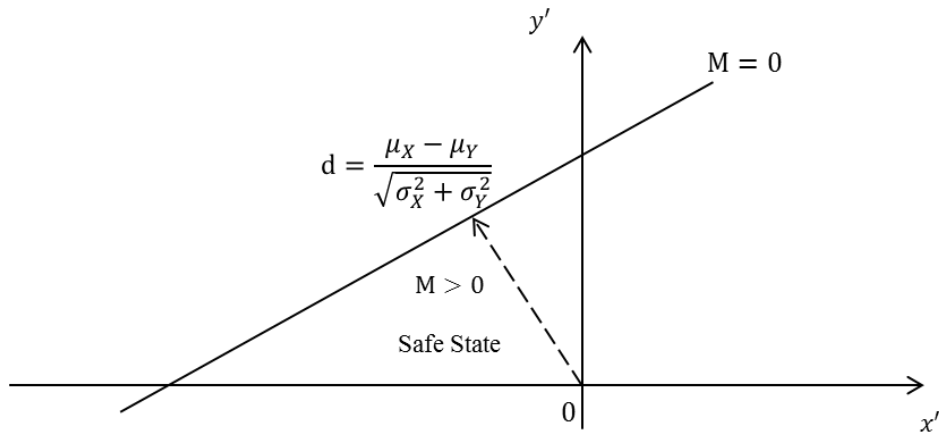
Therefore:

$$M = \sigma_X X' - \sigma_Y Y' + \mu_X - \mu_Y \quad (2.20)$$

In Figure 5, the distance from the origin to line  $M = 0$  is

$$d = \frac{\mu_X - \mu_Y}{\sqrt{\sigma_X^2 + \sigma_Y^2}} \quad (2.21)$$

which is  $\beta$ , the reliability index.



**Figure 5 Safety Margin in reduced variable space [4]**

In reality, reliability problems are usually associated with several different random variables. Thus, in order to generalize reliability problems, a performance function should be defined as [4]:

$$f(\mathbf{X}) = f(X_1, X_2, \dots, X_n) \quad (2.22)$$

where:

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} \quad (2.23)$$

The function above is also named as the state function. It's a function of all the random variables involved in a particular reliability problem. The limit state of system is defined as [4]:

$$f(\mathbf{X}) = 0 \quad (2.24)$$

So the safe state and failure state are identified by

$$[f(\mathbf{X}) > 0] = \text{safe state} \quad (2.25)$$

$$[f(\mathbf{X}) < 0] = \text{failure state} \quad (2.26)$$

Thus:

$$p_f = P(f(\mathbf{X}) < 0) \quad (2.27)$$

$$p_s = P(f(\mathbf{X}) > 0) \quad (2.28)$$

Each random variable can be transformed into a new space as:

$$X'_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \quad (2.29)$$

These random variables are called reduced variables. So the system limit state function can be written as:

$$f(\mathbf{X}) = g(X_1, X_2, \dots, X_n) \quad (2.30)$$

$$= f(X'_1 \sigma_{X_1} + \mu_{X_1}, X'_2 \sigma_{X_2} + \mu_{X_2}, \dots, X'_n \sigma_{X_n} + \mu_{X_n}) = 0$$

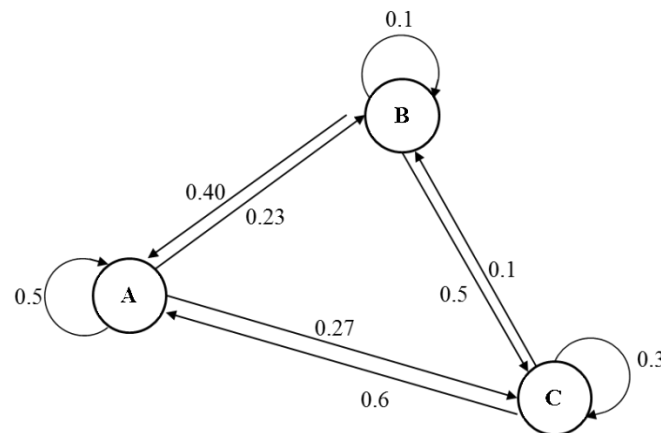
Supposed there is a point  $X'$  on failure surface  $g(\mathbf{X}) = 0$ , the distance  $D$  should be:

$$D = \sqrt{X_1'^2 + X_2'^2 + \dots + X_n'^2} = \sqrt{X'^T X'} \quad (2.31)$$

Thus the reliability is the distance from the origin to limit state surface [4].

## 2.4 Basic concepts of Markov chain

In 1907, A. A. Markov began the study of an important type of process. In this process, the outcome of a given experiment can affect the outcome of the next experiment. This type of process is called Markov chain [6]. The probabilities that the system is at in certain state are represented by a vector  $S = \{p_1, p_2, \dots, p_r\}$ . The system can move from one state to another after a period of time. The probability to move from current state  $i$  to another state  $j$  is defined as the transition probability which is denoted as  $p_{ij}$ [6]. The probability that the system remains at the same state is denoted as  $p_{ii}$ . The figure below shows a system which has only three possible states A, B and C.



**Figure 6 Transition among state A, state B and state C**

The associated probabilities for each step are listed in the matrix below.

$$\mathbf{P} = \begin{bmatrix} 0.5 & 0.4 & 0.6 \\ 0.23 & 0.1 & 0.1 \\ 0.27 & 0.5 & 0.3 \end{bmatrix} \quad (2.32)$$

For example, in this matrix [6]

$$p_{11} = P(\text{system stays at state A}) \quad (2.33)$$



$$p_{12} = P(\text{move from state A to state B}) \quad (2.34)$$

$$p_{13} = P(\text{move from state A to state C}) \quad (2.35)$$

This matrix is named as transition matrix for a Markov chain. Assume that initially the probability that the system is in state A is  $p_{0A}$ , the probability that system is in state B is  $p_{0B}$  and the probability for state C is  $p_{0C}$ . Thus, at the end of one transition, the probabilities for each state are:

$$p_{1A} = p_{11}p_{0A} + p_{12}p_{0B} + p_{13}p_{0C} \quad (2.36)$$

$$p_{1B} = p_{21}p_{0A} + p_{22}p_{0B} + p_{23}p_{0C} \quad (2.37)$$

$$p_{1C} = p_{31}p_{0A} + p_{32}p_{0B} + p_{33}p_{0C} \quad (2.38)$$

written in a matrix form:

$$\begin{bmatrix} p_{1A} \\ p_{1B} \\ p_{1C} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} p_{0A} \\ p_{0B} \\ p_{0C} \end{bmatrix} \quad (2.39)$$

The vector  $\mathbf{S}(n)$  that contains probabilities for every state after  $n$  transitions is name as the state distribution vector. Generally speaking,

$$\mathbf{S}(n) = \mathbf{P} \mathbf{S}(n - 1) \quad (2.40)$$

Notice that,

$$\sum_{i=1}^n p_{ij} = 1 \quad (2.41)$$

And,

$$p_{ij} < 1 \quad (2.42)$$

for all  $i$  and  $j$ . This transition matrix  $\mathbf{P}$  always has an eigenvalue of 1, and all the other eigenvalues are smaller than 1 [6]. Assume that the eigenvector matrix is denoted as  $\mathbf{V}$  and the eigenvalue matrix is denoted as  $\mathbf{D}$ , so [6]

$$\mathbf{P} = \mathbf{V}\mathbf{D}\mathbf{V}^{-1} \quad (2.43)$$

$\mathbf{D}$  is a diagonal matrix, thus [6]:

$$\begin{aligned} \mathbf{S}(n) &= \mathbf{P}^n \mathbf{S}(0) \\ &= (\mathbf{V}\mathbf{D}\mathbf{V}^{-1})^n \mathbf{S}(0) \\ &= \mathbf{V}\mathbf{D}^n\mathbf{V}^{-1} \mathbf{S}(0) \end{aligned} \quad (2.44)$$

When  $n \rightarrow \infty$ ,  $\mathbf{D}$  is tending to be a zero matrix except for the term 1. Thus [6]

$$\lim_{n \rightarrow \infty} \mathbf{S}(n) = \mathbf{S}_{stationary} \quad (2.45)$$

That is to say, when  $n \rightarrow \infty$ , the state distribution vector is infinitely approaching a certain vector, which is name as the stationary vector. This stationary vector is also a measurement of the performance of a truss structure.

The states of a truss structure over its lifetime can be modeled by Markov chain, and this will be discussed in the following chapters.

## 2.5 Basic concepts about system analysis

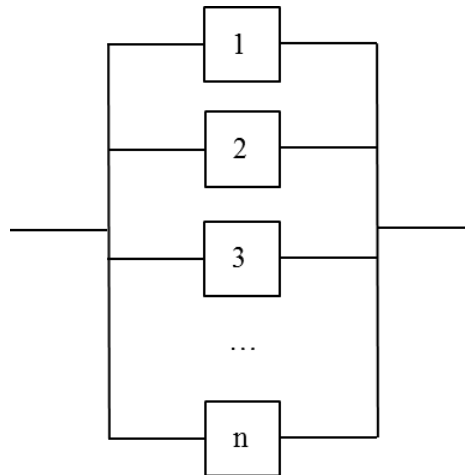
For a bar component  $i$ , its working condition is named as its state  $x_i$ , which has only two possible values: 0 and 1 [10].

$$x_i = \begin{cases} 0 & \text{if component } i \text{ is failed} \\ 1 & \text{if component } i \text{ is functional} \end{cases} \quad (2.46)$$

The states of  $n$  components form a vector  $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$ . The working condition of the overall truss structure can be functional or failed, which is determined by component states, so it is a function of vector  $\mathbf{x}$  [10].

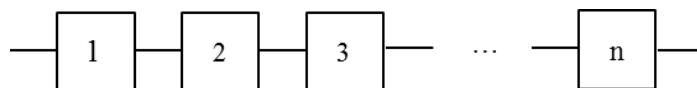
$$\phi(\mathbf{x}) = \begin{cases} 0 & \text{if system is failed} \\ 1 & \text{if system is functional} \end{cases} \quad (2.47)$$

For a system, there are two types of basic arrangements: the parallel arrangement and the series arrangement. Figure 7 shows the block diagram of a parallel system.



**Figure 7 Block diagram of parallel system [10]**

A parallel system will not fail until all of its components fail, while a series system will fail as soon as any one of its components fails. The block diagram of a series system is shown in the figure below.



**Figure 8 Block diagram of series system [10]**

For a series system, the system state function is [10]:

$$\phi(\mathbf{x}) = 1 - \prod_{i=1}^n (1 - x_i) \quad (2.48)$$

Correspondingly, for a parallel system [10]:

$$\phi(\mathbf{x}) = \prod_{i=1}^n x_i \quad (2.49)$$

The probability that the component  $i$  is functioning is

$$p_i = P(X_i = 1) \quad (2.50)$$

Correspondingly, the system reliability is [10]

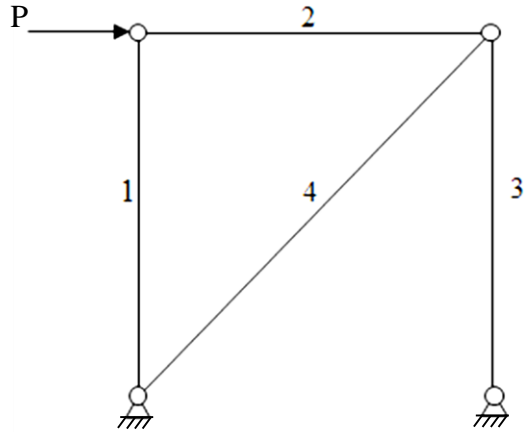
$$r = P[\phi(\mathbf{x}) = 1] \quad (2.51)$$

## 2.7 Case study: reliability analysis of a 4-bar truss

Figure 9 shows a simple truss structure with four bar components. It is carrying only one external load. The components are labeled from one to four. The cross section areas, the material properties and the value of the load are all random variables. Their parameters are listed in Table 1.

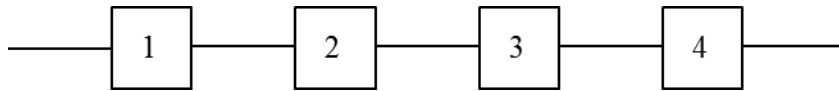
**Table 1 Properties of 4-bar truss structure**

Item	Mean	Std.	Type of Distribution
A1( $m^2$ )	3.20E-04	3.00E-05	Lognormal
A2( $m^2$ )	3.20E-04	3.00E-05	Lognormal
A3( $m^2$ )	3.20E-04	3.00E-05	Lognormal
A4( $m^2$ )	4.80E-04	4.00E-05	Lognormal
E(Pa)	2.35E+08	2.35E+06	Lognormal
Load(N)	5.50E+04	5.50E+03	Lognormal
Ultimate Stress(Pa)	5.50E+04	5.50E+03	Lognormal



**Figure 9 A 4 bar truss**

This structure is statically determinate, so can be modeled as a series system (the structure will fail if any one of the four bar components fails). The block diagram of this truss structure is shown below:



**Figure 10 Block diagram of simple truss with no redundancy**

The limit state functions of the bar components are:

$$g_1 = \sigma_{ultimate1}A_1 \quad (2.52)$$

$$g_2 = \sigma_{ultimate2}A_2 - P \quad (2.53)$$

$$g_3 = \sigma_{ultimate3}A_3 - P \quad (2.54)$$

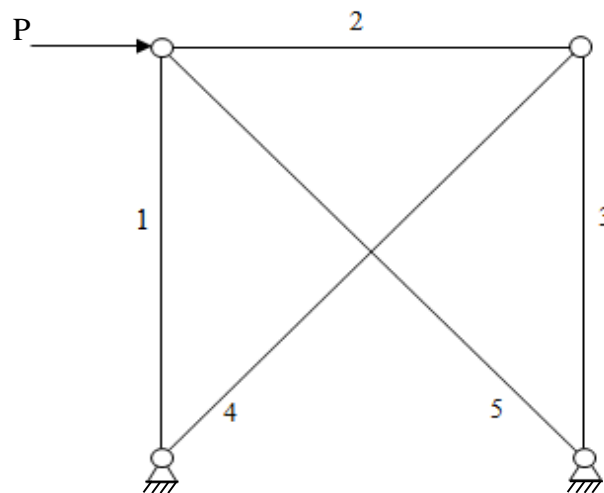
$$g_4 = \sigma_{ultimate4}A_4 - \sqrt{2}P \quad (2.55)$$

The results of component reliability analysis are listed in the table below.

**Table 2 Failure probabilities of 4-bar truss system**

Component	Probability of failure
1	0
2	0.0023
3	0.0023
4	0.0018

Figure 11 shows a 5-bar truss. Even if only one additional bar component is added to the original 4-bar truss structure, its complexity will increase a lot. The new truss will not fail until two of its five bar components fail, so it can be modeled as a combination of series sub-systems and parallel sub-systems. All of the possible two-bar failure modes are listed in Table 4.



**Figure 11 A 5-bar truss system**

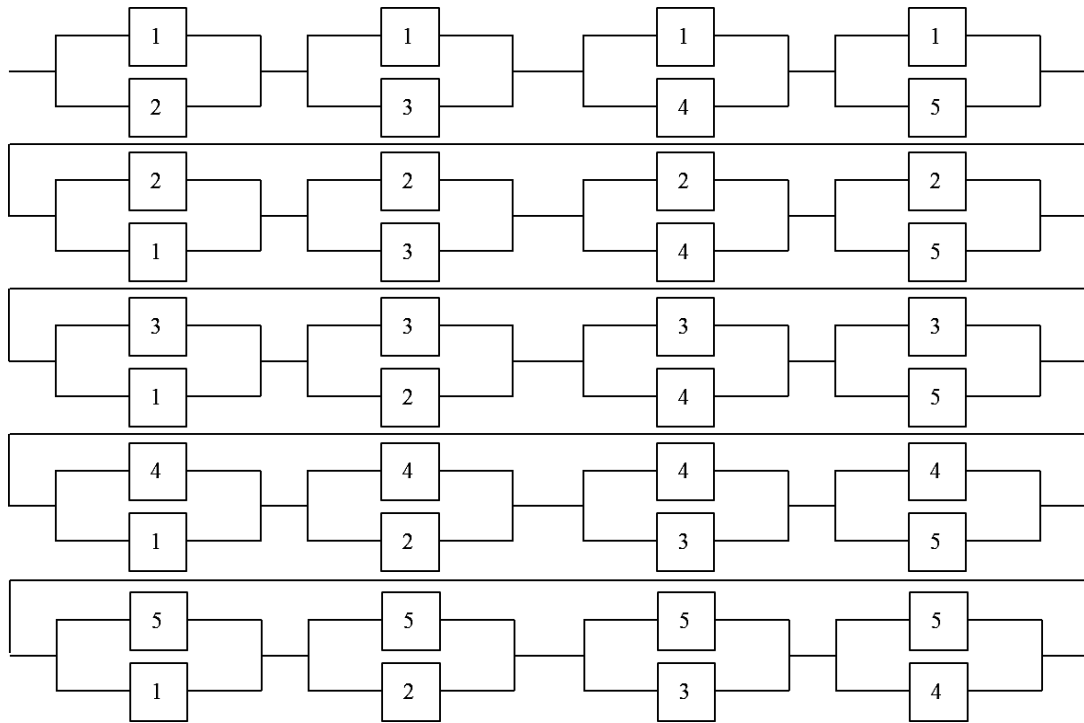
**Table 3 Properties of 5-bar truss structure**

Item	Mean	Std.	Type of Distribution
A1( $m^2$ )	3.20E-04	3.00E-05	Lognormal
A2( $m^2$ )	3.20E-04	3.00E-05	Lognormal
A3( $m^2$ )	3.20E-04	3.00E-05	Lognormal
A4( $m^2$ )	4.80E-04	4.00E-05	Lognormal
A5( $m^2$ )	4.80E-04	4.80E-05	Lognormal
E(Pa)	2.35E+08	2.35E+06	Lognormal
Ultimate Stress(Pa)	5.50E+04	5.50E+03	Lognormal

**Table 4 Failure sequence of the 5-bar truss structure**

First failed component	Second failed component
1	2
	3
	4
	5
2	1
	3
	4
	5
3	1
	2
	4
	5
4	1
	2
	3
	5
5	1
	2
	3
	4

The block diagram of this 5-bar truss system is shown in Figure 12. Compared with Figure 10, this new block diagram is much more complex even if only one bar component is added to the original 4-bar truss structure. In reality, a truss structure usually has more than 4 bar components. Hence, it is not easy to analyze the reliability of a truss structure that has a large number of bar components.



**Figure 12 Block diagram for structure with one degree of static indeterminacy**

## 2.6 Poisson process and exponential distribution

The Poisson process follows the following assumptions[4]:

1. The event can occur randomly at any instant of time during a period of time
2. The occurrence of an event in a given time interval is statistically independent of that in any other nonoverlapping interval.
3. The probability of occurrence of an event in a small interval is proportional to the length of the interval



4. The probability of two or more occurrences in the small interval is negligible (of higher orders of time interval)

The PMF of Poisson process is giving as [4]:

$$P(X_t = x) = \frac{(vt)^x}{x!} e^{-vt} \quad (2.56)$$

$X_t$  is the number of occurrences in the time interval  $(0, t)$ .

For a Poisson process, assume  $T_1$  is the time till the first occurrence of an event. Thus the event  $(T_1 > t)$  [4] means that there is no occurrence of the event during time interval  $(0, t)$ . Thus:

$$P(T_1 > t) = P(X_t = 0) = e^{-vt} \quad (2.57)$$

so, the probability that the event will occur during time interval  $(0, t)$  is

$$P(\text{occur}) = 1 - e^{-vt} \quad (2.58)$$

where  $v$  is the mean occurrence rate.

## Chapter 3

### Component Reliability

Bar components are the most basic structural units for a truss structure. As discussed previously, system reliability is calculated based on component reliability. This chapter discusses the topics of component reliability.

#### 3.1 Matrix analysis and Monte Carlo simulation

Matrix analysis is a numerical method which has long been used for structural analysis. In this study, we assume that the nodes of each component can move in x and y directions, and all the external loads act only on nodes. We also assume that all the steel components are assumed to work in the linear elastic range. If the absolute value of the stress in a bar component exceeds the associated ultimate stress, this bar component will fail.

For a truss structure like this, there exists a linear relation between external loads and nodal displacements. Element stiffness matrices that describe the relation between nodal displacements and nodal forces for each component are formulated by energy based methods. These element stiffness matrices are superimposed together to form the global stiffness matrix  $\mathbf{K}$ , which describes the behavior of the overall truss structural system.

$$\mathbf{k}_i^g = \mathbf{T}^T \mathbf{k}_i \mathbf{T} \quad (3.1)$$

$$\mathbf{K} = \sum \mathbf{k}_i^g \quad (3.2)$$

Here  $\mathbf{k}_i$  is the element stiffness matrix under the local coordinate system.  $\mathbf{k}_i^g$  is the element stiffness matrix under the global coordinate and  $\mathbf{T}$  is the transformation matrix. The basic formulation of matrix analysis for truss structure is

$$\mathbf{Ku} = \mathbf{F} \quad (3.3)$$

In equation (3.3),  $\mathbf{K}$  stands for the global stiffness matrix.  $\mathbf{u}$  represents the nodal displacements at each node.  $\mathbf{F}$  is the external loads acting on each degree of freedom, which is already known

$$\mathbf{u} = \mathbf{K}^{-1}\mathbf{F} \quad (3.4)$$

By inverting global stiffness matrix and pre-multiplying it on both sides of equation (3.3), we get displacements of each node. Finally, deformations and stress distributions in each bar component are calculated based on nodal displacements.

The basic idea of Monte Carlo simulation is to simulate the stochastic experiments by numerical methods. For example, a computer program can generate a set of samples of a random variable which satisfies a certain type of distribution. These samples are plugged into the matrix analysis procedure which will generate samples of stress distributions in each component. The working condition of a bar component can be identified by checking its stress. If the absolute value of stress in a bar component exceeds the ultimate stress, that particular bar component is identified to be failed. During each loop, some components are identified to be failed while others are still functional. Hence, the survival probability of the bar component equals the number of failed samples divided by the total number of samples. As the total number of samples is increasing, the results are tending to approach the exact theoretic values.

The first step of the MCS procedure is to generate samples for each random variable. Once all the samples have been generated by Matlab code, the reliabilities of each component will be calculated. The component reliabilities of truss structure

shown in Figure 9 are calculated using Monte Carlo simulation procedure, and the results are listed in Table 5.

In Table 5, the second row shows the failure probabilities of each bar component in the 4-bar truss structure calculated by MCS method, and the third row contains exact values of failure probabilities as discussed in Chapter 2. These two sets of results match each other well. The results calculated by MCS method are good estimations of reliabilities of each bar component (with 10000 samples). Compared with limit state function method, Monte Carlo simulation is more suitable for automatic numerical calculation. The main disadvantage of this method is that during the simulation procedure, the global stiffness matrix  $\mathbf{K}$  is required to be inverted many times. Under the Matlab programming environment, inverting a large-sized matrix takes a very large computational effort.

### 3.3 Perturbation method

The fundamental idea of the perturbation method is based on the Taylor expansion of the global stiffness matrix. The global stiffness matrix and the displacements are functions of random variables.

$$\mathbf{K}(\boldsymbol{\alpha})\mathbf{u}(\boldsymbol{\alpha}) = \mathbf{F} \quad (3.5)$$

In equation (3.5),  $\boldsymbol{\alpha}$  represents the vector of all the random variables. Hence, equation (3.5) can be written as:

$$(\mathbf{K}_0 + \mathbf{K}_i(\boldsymbol{\alpha}))\mathbf{u}(\boldsymbol{\alpha}) = \mathbf{F} \quad (3.6)$$

Here  $\mathbf{K}_0$  is mean stiffness matrix, and matrix  $\mathbf{K}_i(\boldsymbol{\alpha})$  is

$$\mathbf{K}_i(\boldsymbol{\alpha}) = \sum_{i=1}^{Nr} \alpha_i \frac{\partial}{\partial \alpha_i} \mathbf{K}_i(\boldsymbol{\alpha})$$

$$+ \sum_{i=1}^{Nr} \sum_{j=1}^{Nr} \alpha_i \alpha_j \frac{\partial^2}{\partial \alpha_i \partial \alpha_j} \mathbf{K}_i(\boldsymbol{\alpha}) + \dots \quad (3.7)$$

For the same reason,  $\mathbf{u}(\boldsymbol{\alpha})$  can be expressed as

$$\begin{aligned} \mathbf{u}(\boldsymbol{\alpha}) &= \mathbf{u}_0 + \sum_{i=1}^{Nr} \alpha_i \frac{\partial}{\partial \alpha_i} \mathbf{u}(\boldsymbol{\alpha}) \\ &+ \sum_{i=1}^{Nr} \sum_{j=1}^{Nr} \alpha_i \alpha_j \frac{\partial^2}{\partial \alpha_i \partial \alpha_j} \mathbf{u}(\boldsymbol{\alpha}) + \dots \end{aligned} \quad (3.8)$$

With equation (3.6), (3.7) and (3.8), we have

$$\left( \mathbf{u}_0 + \sum_{i=1}^{Nr} \alpha_i \frac{\partial}{\partial \alpha_i} \mathbf{u}(\boldsymbol{\alpha}) \right) \left( \mathbf{K}_0 + \sum_{i=1}^{Nr} \alpha_i \frac{\partial}{\partial \alpha_i} \mathbf{K}_i(\boldsymbol{\alpha}) \right) = \mathbf{F} \quad (3.9)$$

So

$$\begin{aligned} \mathbf{K}_0 \mathbf{u}_0 + \left( \sum_{i=1}^{Nr} \alpha_i \frac{\partial}{\partial \alpha_i} \mathbf{K}_i(\boldsymbol{\alpha}) \right) \mathbf{u}_0 + \mathbf{K}_0 \left( \sum_{i=1}^{Nr} \alpha_i \frac{\partial}{\partial \alpha_i} \mathbf{u}(\boldsymbol{\alpha}) \right) \\ + \left( \sum_{i=1}^{Nr} \alpha_i \frac{\partial}{\partial \alpha_i} \mathbf{K}_i(\boldsymbol{\alpha}) \right) \left( \sum_{i=1}^{Nr} \alpha_i \frac{\partial}{\partial \alpha_i} \mathbf{u}(\boldsymbol{\alpha}) \right) = \mathbf{F} \end{aligned} \quad (3.10)$$

For each  $\alpha_i$ ,

$$\mathbf{K}_0 \left( \sum_{i=1}^{Nr} \alpha_i \frac{\partial}{\partial \alpha_i} \mathbf{u}(\boldsymbol{\alpha}) \right) + \left( \sum_{i=1}^{Nr} \alpha_i \frac{\partial}{\partial \alpha_i} \mathbf{K}_i(\boldsymbol{\alpha}) \right) \mathbf{u}_0 = \mathbf{0} \quad (3.11)$$

Thus

$$\mathbf{K}_0 \frac{\partial}{\partial \alpha_i} \mathbf{u}(\boldsymbol{\alpha}) + \frac{\partial}{\partial \alpha_i} \mathbf{K}(\boldsymbol{\alpha}) \mathbf{u}_0 = 0 \quad (3.12)$$

and,

$$\frac{\partial}{\partial \alpha_i} \mathbf{u}(\boldsymbol{\alpha}) = -\mathbf{K}_0^{-1} \frac{\partial}{\partial \alpha_i} \mathbf{K}(\boldsymbol{\alpha}) \mathbf{u}_0 \quad (3.13)$$

Finally,

$$\mathbf{u}(\boldsymbol{\alpha}) = \left[ \mathbf{I} - \left( \sum_{i=1}^{Nr} \alpha_i \mathbf{K}_0^{-1} \frac{\partial}{\partial \alpha_i} \mathbf{K}(\boldsymbol{\alpha}) \right) \right] \mathbf{u}_0 \quad (3.14)$$

It is clear that in equation (3.14) the global stiffness matrix needs only to be inverted once, which makes numerical computation much faster.

**Table 5 Results of reliability analysis**

Bar Element	1	2	3	4
Probability of failure by Perturbation method	0	0.0019	0.0021	0.0015
Probability of failure by MCS method	0	0.0023	0.0022	0.0017
Probability of failure (exact value)	0	0.0023	0.0023	0.0018

The results calculated by perturbation method are listed in Table 5. From this table we can see that the Monte Carlo simulation may offer reliable results for probability problems, but its computational inefficiency is a disadvantage. On the other hand, the perturbation method is more computational efficient. However, it is not as accurate as MCS method. As a summary, the perturbation method is more suitable for a brief estimation, while MCS is good for comprehensive and accurate calculation.

## Chapter 4

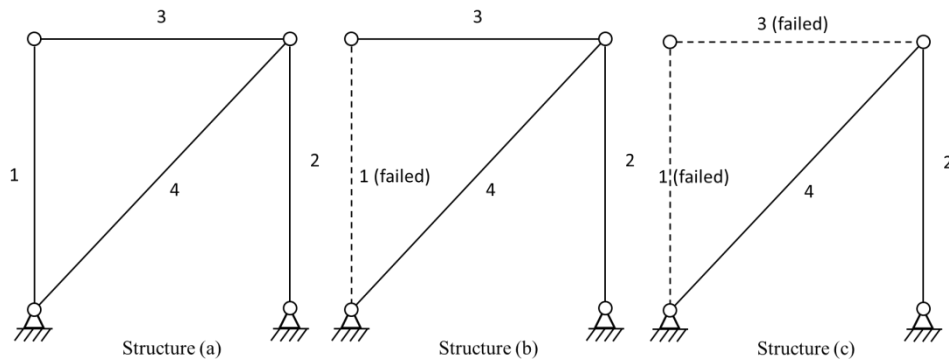
### Time-variant reliability analysis of trusses

#### 4.1 Introduction

As mentioned in Chapter 3, there are only two possible states for a component: the failure state or the functional state. Thus the state for a bar component can be defined as a binary number

$$C_i = \begin{cases} 1 & \text{if component } i \text{ is functional} \\ 0 & \text{if component } i \text{ is failed} \end{cases} \quad (4.1)$$

The example in Figure 13 illustrates the concept of component state in a truss structural system.



**Figure 13 An example of component state**

**Table 6 Component state in each structure**

Index of component	Component state of structure (a)	Component state of structure (b)	Component state of structure (c)
1	1	0	0
2	1	1	1
3	1	0	1
4	1	1	1

As discussed in Chapter 3, the probability of safety of a component can be calculated by numerical method, and it is denoted as:

$$ps_{oi} = P(C_i = 1) \quad (4.2)$$

and the failure probability of component  $i$  is

$$pf_{oi} = P(C_i = 0) \quad (4.3)$$

#### 4.2 Poisson process and discretization of time

Let's consider only one bar component of a truss structure. This particular component may fail at any point in time. It is possible for this component to fail after one month, or it is also possible for it to fail after ten years. The time before the failure of the component is also a random variable. According to the theory of probability, this kind of random variable can be described by a Poisson process.

Suppose there is a period of time  $T$  which is discretized into  $n$  small time intervals  $t_i$  ( $i = 1, 2, 3, \dots, n$ ). In each of the small time intervals, an event may occur or may not occur. That is to say, there are only two possibilities, the occurrence of the event and the nonoccurrence of the event. Assume the occurrence in time interval  $t_i$  will not influence the occurrence in time interval  $t_j$  ( $i \neq j$ ). Suppose that the probability of occurrence in each small time interval is a constant denoted as  $p_i$ . During the total amount of time  $T$ , the probability that the event occurs  $x$  times should be: [4]

$$P(X = x \text{ in } T) = \binom{n}{x} p_i^x (1 - p_i)^{n-x} \quad (4.4)$$

This kind of distribution is named as Binomial distribution.  $\lambda$  is the mean occurrence time during time  $T$ . According to the assumptions above

$$p_i = \frac{\lambda}{n} \quad (4.5)$$

thus equation (4.4) becomes



$$P(X = x \text{ in } T) = \binom{n}{x} \frac{\lambda^x}{n} \left(1 - \frac{\lambda}{n}\right)^{n-x} \quad (4.6)$$

We can refine the discretization of time period  $T$  by increasing the total number of small time intervals  $t_i$ , that is to say  $n \rightarrow \infty$ . If so,

$$P(X = x \text{ in } T) = \lim_{n \rightarrow \infty} \binom{n}{x} \frac{\lambda^x}{n} \left(1 - \frac{\lambda}{n}\right)^{n-x} \quad (4.7)$$

where

$$\begin{aligned} & \binom{n}{x} \frac{\lambda^x}{n} \left(1 - \frac{\lambda}{n}\right)^{n-x} \\ &= \frac{n(n-1)}{n} \dots \frac{(n-x+1)\lambda^n}{n} \frac{1}{x!} \left(1 - \frac{\lambda}{n}\right)^n \left(1 - \frac{\lambda}{n}\right)^{-x} \end{aligned} \quad (4.8)$$

and

$$\lim_{n \rightarrow \infty} \left(1 - \frac{\lambda}{n}\right)^n = e^{-\lambda} \quad (4.9)$$

and

$$\lim_{n \rightarrow \infty} \left(1 - \frac{\lambda}{n}\right)^{-x} = 1 \quad (4.10)$$

so

$$P(X = x \text{ in } T) = \lim_{n \rightarrow \infty} \binom{n}{x} \frac{\lambda^x}{n} \left(1 - \frac{\lambda}{n}\right)^{n-x} = \frac{\lambda^x}{x!} e^{-\lambda} \quad (4.11)$$

This kind of distribution is call Poisson distribution, and the process that satisfies the Poisson distribution is named Poisson process. For a Poisson process, the mean occurrence number during time  $T$  is normally estimated based on observations, and it is often written as

$$\lambda = \nu T \quad (4.12)$$

where  $\nu$  is the mean occurrence rate.

Assume the time before the first occurrence of the event is  $T_1$ . Thus,  $(T_1 > T)$  means that the event will not occur during the time period  $T$ . Hence,

$$P(T_1 > T) = P(X = 0 \text{ in } T) = \frac{0^n}{0!} e^{-\lambda} = e^{-\lambda} \quad (4.13)$$

so the probability that the event will occur during time period of  $T$  is

$$P(\text{occur in } T) = P(T_1 \leq T) = 1 - P(T_1 > T) = 1 - e^{-\lambda} \quad (4.14)$$

With (4.12) and (4.14),

$$P(\text{occur in } T) = 1 - e^{-\nu T} \quad (4.15)$$

Let's consider the following example. Suppose there are many balls in a box. All the balls are in one of the two colors, yellow or blue. Assume that if we randomly pick a ball from the box

$$P(\text{ball is blue}) = p_b \quad (4.16)$$

$$P(\text{ball is yellow}) = 1 - p_b \quad (4.17)$$

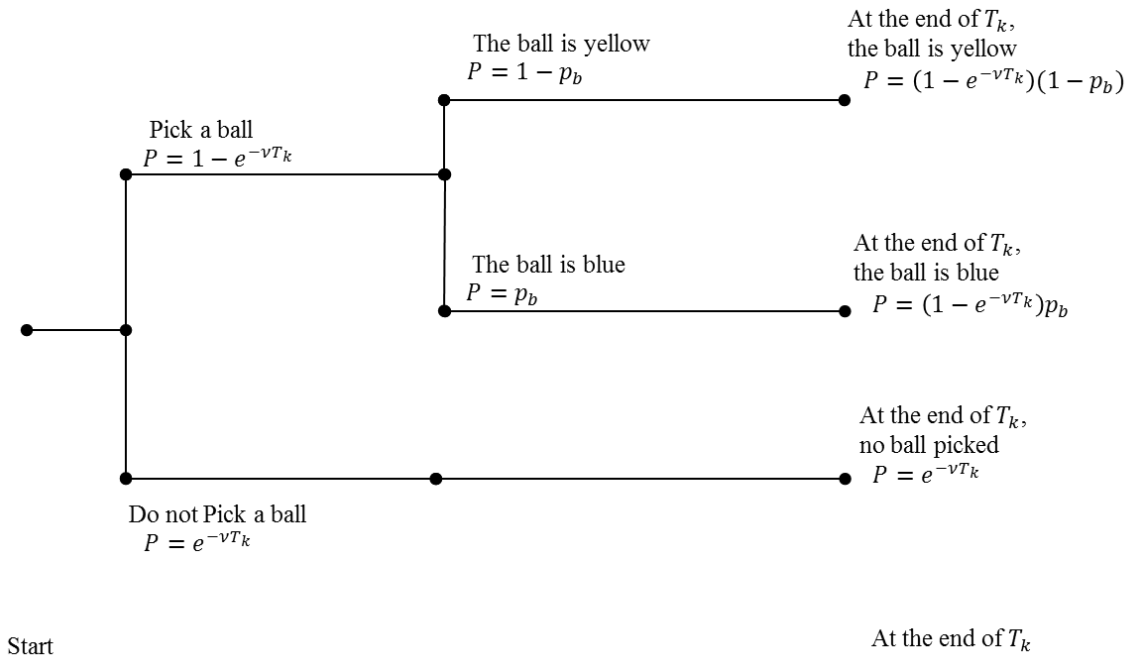
During a time period of  $T_k$ , we may choose or may not choose a ball from the box.

Let's name the event of picking up a ball as event C. Thus, the occurrence of event C during the time period of  $T_k$  can be modeled by equation (4.15). Therefore

$$P(\text{do pick a ball}) = 1 - e^{-\nu T_k} \quad (4.18)$$

$$P(\text{do not pick a ball}) = e^{-\nu T_k} \quad (4.19)$$

Hence, all the possible realizations of this process are listed in Figure 14.



**Figure 14 Sequence of random realizations over  $T_k$**

Therefore, at the end of  $T_k$ , the probability that the ball is yellow is

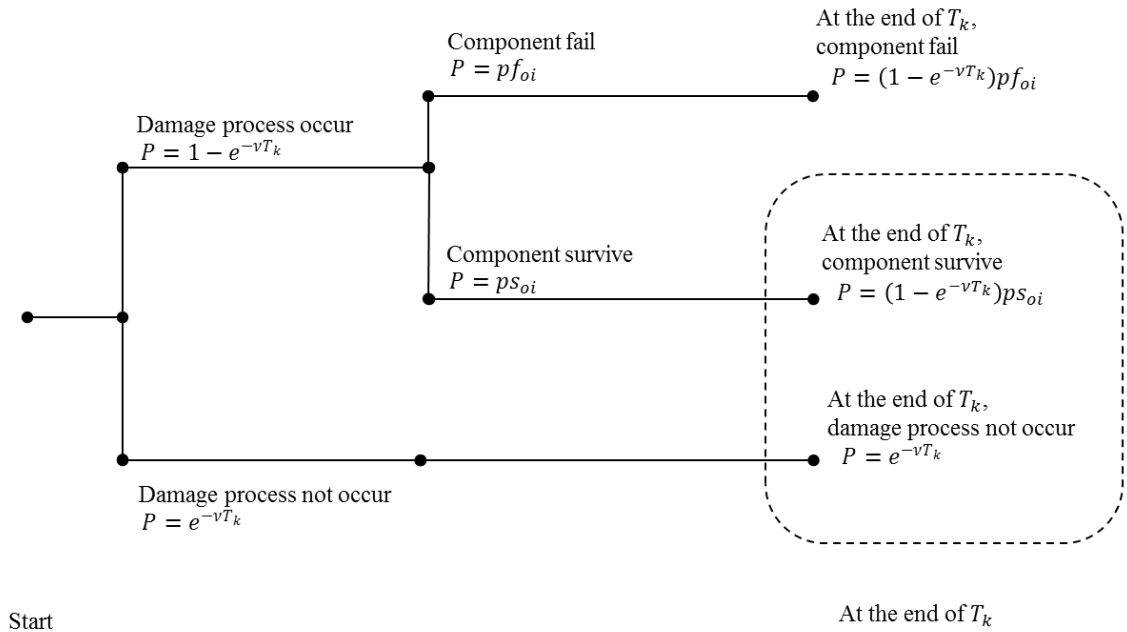
$$P(\text{ball is yellow when } t = T_k) = (1 - e^{-vT_k})(1 - p_b) \quad (4.20)$$

and the probability that the ball is blue is

$$P(\text{ball is blue when } t = T_k) = (1 - e^{-vT_k}) p_b \quad (4.21)$$

This process can be repeated after the time  $T_k$ . The mean occurrence rate  $v$  remains the same. Looping this process  $n$  times, we can get the probability that the ball is yellow at time  $nT_k$ .

The process discussed above is an appropriate analogy of the behavior of the bar component. Assume the bar component is fully functional initially. During a time period of  $T_k$ , the condition of damage of the bar component may or may not be satisfied. Then, if the damage occurs, the component may survive or may not survive. The diagram of this process is shown in Figure 15.



**Figure 15 Sequence of random realizations over  $T_k$**

Hence, after  $T_k$ , the probability that the component is at failure state is:

$$pf_i = (1 - e^{-vT_k})pf_{oi} \quad (4.22)$$

$v$  is determined based on experience and it is a constant value.  $T_k$  is the time period considered. The probability that the component will survive after  $T_k$  is

$$ps_i = 1 - pf_i \quad (4.23)$$

The mean occurrence rate of this process depends on both environment and the component itself. Therefore, for a certain type of truss structure under a certain kind of environment,  $(1 - e^{-vT_k})$  is a constant. Herein the term  $(1 - e^{-vT_k})$  is estimated to be 0.9.

### 4.3 System states

The state of each component is defined as  $C_i$ . Hence, the states for all the components form the vector:

$$(C_1, C_2, C_3, C_4, \dots, C_i)^T \quad (4.24)$$

Since there are only two possible values (0 and 1) for each  $C_i$ , by joining all the digits together, it forms a binary number  $B$ :

$$B = \underline{C_1 C_2 C_3 C_4 \dots C_i} \quad (4.25)$$

This binary number can be transformed to a decimal number as

$$D'_j = C_1 \times 2^{i-1} + C_2 \times 2^{i-2} + C_3 \times 2^{i-3} + C_4 \dots + C_i \times 2^{i-i} \quad (4.26)$$

Since in Matlab index of a sequence starts from 1, so the  $j^{\text{th}}$  system state is represented by

$$D_j = D'_j + 1 \quad (4.27)$$

Here is an example of system state representation for a 5-bar truss structure:

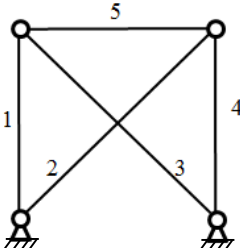
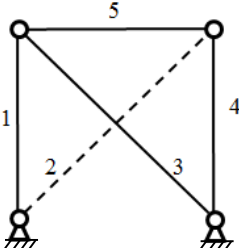
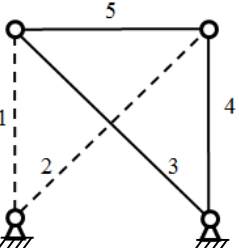
	Fully functional system state (a)	System with component 2 failed (b)	System with component 1 and 2 failed (c)
Truss structure			
Binary Representation	11111	10111	00111
Decimal $D'$	31	23	7
Decimal $D=D'+1$	32	24	8
System failed or not	No	No	YES
System condition	1	1	0

Figure 16 Example of system state representation

In Figure 16 (a), all the bar components are fully functional, so the component states are (1, 1, 1, 1, 1), and this system state can be represented by decimal number 32. For the same reason, system state in Figure 16 (b) can be represented by decimal number 24 and system state in Figure 16 (c) can be represented by decimal number 8. There's no bar component broken in structure (a), so it is fully functional. There is only one bar component failed in structure (b), so it is also functional. However, in structure (c), the failure of component 1 and component 2 makes the truss structure a mechanism.

The survival probabilities of all the bar components after  $T_k$  can be written in a vector form as

$$(ps_1, ps_2, ps_3, \dots, ps_n) \quad (4.28)$$

and the corresponding failure probabilities can be written as

$$(pf_1, pf_2, pf_3, \dots, pf_n) \quad (4.29)$$

where

$$pf_i = 1 - ps_i \quad (4.30)$$

It is important to notice that the survival probabilities for a certain bar component in different system states are different.

#### 4.4 Transition probability

After a period of time  $T_k$ , the system can move from one state to another. The probability that system moves from one state to another can be regarded as a connection between these two states. The transition among different system states can thus form a network.

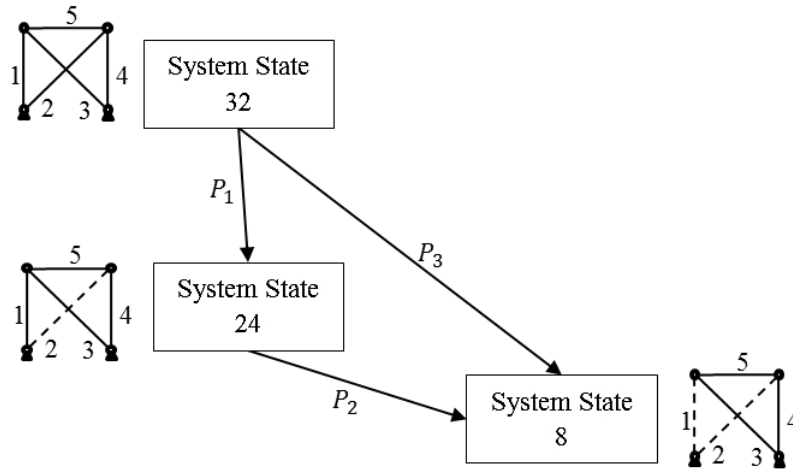
Consider the 5-bar truss structural system in Figure 17 as an example. Figure 17 demonstrates how the system states are connected. If the bar component 2 in state 32 fails and all the other components are functional after a period of time, the system will move from state 32 to state 24.

Recall that the survival probabilities of each bar component over  $T_k$  are

$$(ps_1(32), ps_2(32), ps_3(32), ps_4(32), ps_5(32)) \quad (4.31)$$

, and the failure probability of bar component 2 over  $T_k$  is

$$(1 - ps_2(32)) \quad (4.32)$$



**Figure 17 Transition between system states**

Thus, assuming statistical independence among failure modes, the probability that connects system state 32 and system state 24 should be

$$P_1 = P_{(32,24)} = (1 - ps_2(32)) \prod_{i=1,3,4,5} ps_i(32) \quad (4.33)$$

, and

$$P_2 = P_{(32,8)} = (1 - ps_1(32))(1 - ps_2(32)) \prod_{i=3,4,5} ps_i(32) \quad (4.34)$$

For the same reason, the survival probabilities of each bar component over  $T_k$  for system state 24 are

$$(ps_1(24), ps_2(24), ps_3(24), ps_4(24), ps_5(24)) \quad (4.35)$$

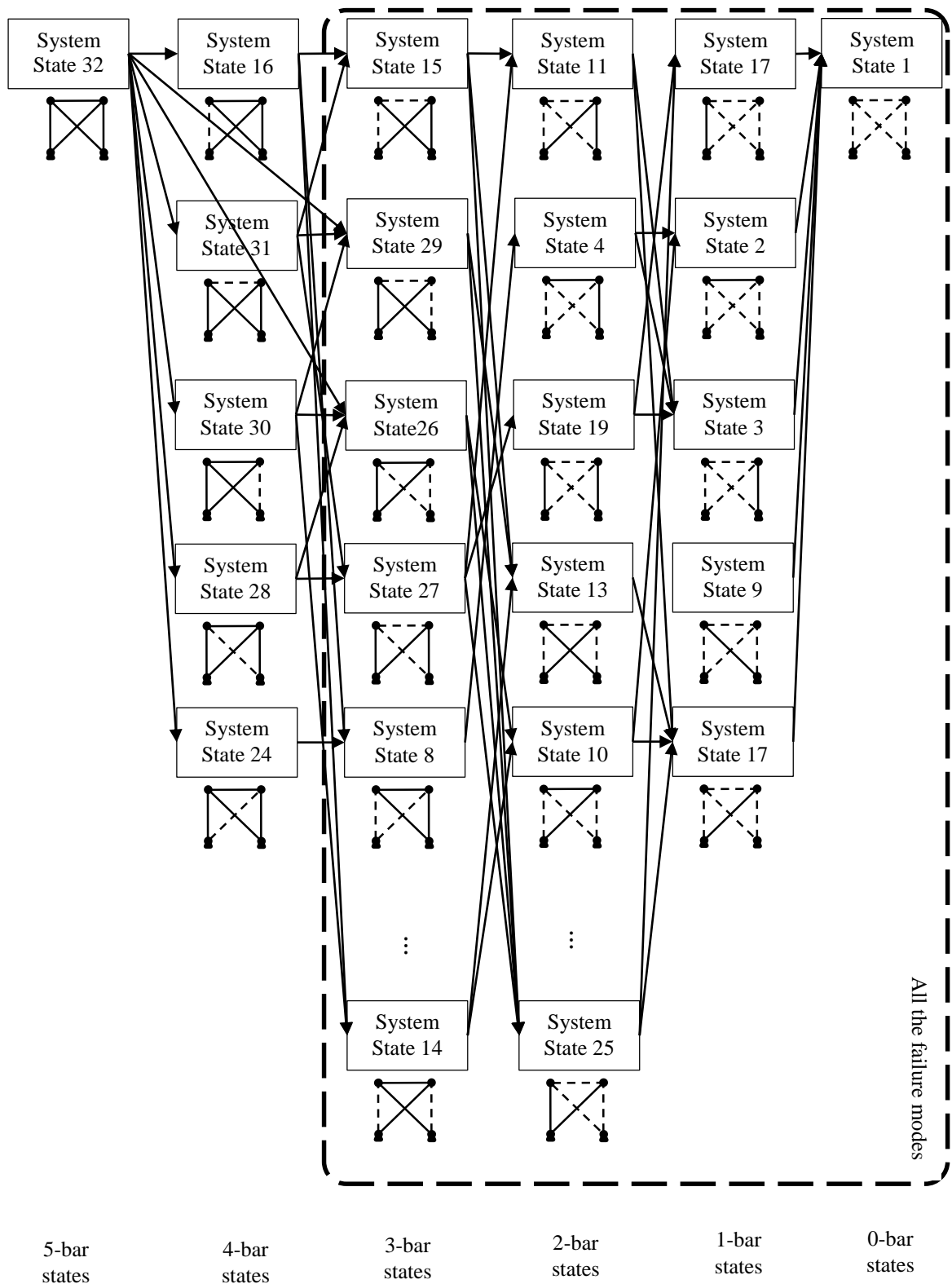
, so the transition probability from state 24 to state 8 is

$$P_3 = P_{(24,8)} = (1 - ps_i(24)) \prod_{i=3,4,5} ps_i(24) \quad (4.36)$$

The method illustrated above can be used to determine the transition probabilities from one state to another. Additionally, in Figure 17, there are two different paths to move from the state 32 to the state 8. The first one starts from state 32, goes to state 24 and ends with state 8. The second path connects state 32 and state 8 directly. Generally speaking, the path that connects state  $a$  and state  $b$  is defined as a path from  $a$  to  $b$ .

It is known that, for a truss system with  $n$  bar components, there are  $2^n$  structural states. The states that can carry the external loads are defined to be functional states, while all the other states are failure states. It is easy to identify failure states and functional states by the matrix analysis method. A truss can no longer carry external loads safely when its global stiffness matrix is singular.





**Figure 18 System states for a 5-bar truss**

In Figure 18, all the possible system states for the 5-bar truss structural system are presented, and they are classified into five different layers based on the number of functional components. Generally speaking, the transition from layer  $i$  to layer  $j$  can be defined as “downward” movement if  $i > j$ , and the layers associated with fewer functional bar components are defined as lower layers. It is possible for a state in a high layer to move downward to a lower layer. However, on the other hand, a system state in a lower layer cannot move upward into a high layer if there’s not maintenance applied.

#### 4.5 Time-variant system reliability

Assume there are  $N$  bar components in a truss structural system, so at a certain time point  $nT_k$ , the system has  $M$  system states

$$M = 2^N \quad (4.37)$$

Let  $P_{step(i,j)}$  represent the probability to move from system state  $i$  to its neighbor state  $j$ , thus

$$P_{step(i,j)} = \prod_{\substack{p=functional \\ bar\ components}} p_{s_p(i)} \prod_{\substack{q=failed\ bar \\ components}} (1 - p_{s_q(i)}) \quad (4.38)$$

There are two kinds of paths: (a) system moves directly from state  $i$  to state  $j$ , which contains only one step, and (b) there are multiple steps from state  $i$  to state  $j$ . For the first situation

$$P_{path(i,j)} = P_{step(i,j)} \quad (4.39)$$

For the second situation

$$P_{path(i,a,b,\dots,d,j)} = P_{step(i,a)}P_{step(a,b)}P_{step(b,\dots)} \dots P_{step(\dots,d)}P_{step(d,j)} \quad (4.40)$$

Thus, the probability to move from state  $i$  to state  $j$  is:

$$P_{(i,j)} = \prod_{\substack{\text{all the possible} \\ \text{paths from } i \text{ to } j}} P_{\text{path}(i,\dots,j)} \quad (4.41)$$

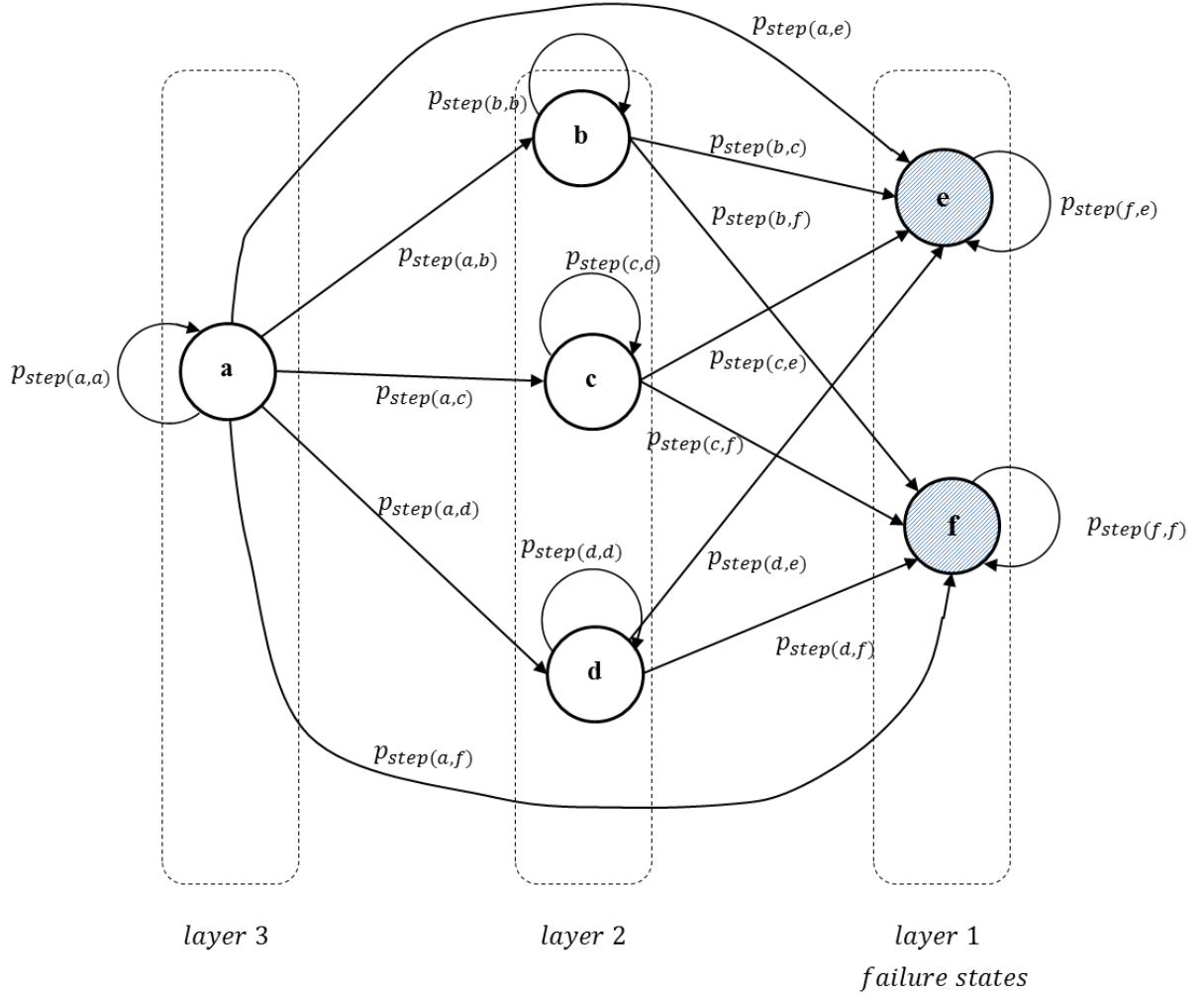
Assume that the system is initially in state  $i$  and the associated probability is  $P_{\text{initial}}$ , so the probability that system is in state  $j$  at a period of time is:

$$P_j = P_{\text{initial}} P_{(i,j)} \quad (4.42)$$

Since system states are mutually exclusive and collectively exhaustive, the summation of probabilities for all the states should be one.

#### 4.6 Markov chain and reliability analysis

Figure 19 shows an example of transitions among six system states. This figure represents the probability map of a truss structure.



**Figure 19 A network of six system states**

In Figure 19, there are 6 system states, and the probability to move from state  $i$  to its neighbor state  $j$  is represented by  $P_{step(i,j)}$ . Suppose that the system is initially in state  $a$ , and the associated probability is  $p_{initial}$ . After a period of time  $T_k$ , the probabilities that system is in a given state is

$$P_a(T_k) = P_{initial} \times P_{step(a,a)} \quad (4.43)$$

$$P_b(T_k) = P_{initial} \times P_{step(a,b)} \quad (4.44)$$

$$P_c(T_k) = P_{initial} \times P_{step(a,c)} \quad (4.45)$$

$$P_d(T_k) = P_{initial} \times P_{step(a,d)} \quad (4.46)$$

$$P_e(T_k) = P_{initial} \times P_{step(a,e)} \quad (4.47)$$

$$P_f(T_k) = P_{initial} \times P_{step(a,f)} \quad (4.48)$$

$P_{step(a,a)}$  represents that the system stays at the initial condition. So the probability of safety of the truss structure at the end time  $T_k$  is:

$$P(\text{safety at } T_k) = P_a(T_k) + P_b(T_k) + P_c(T_k) + P_d(T_k) \quad (4.49)$$

After a period of time  $2T_k$ , each path should contain 2 steps. All the possible paths are listed in Table 9. The associated probabilities are

$$\begin{aligned} P_a(2T_k) &= P_{initial} P_{step(a,a)} P_{step(a,a)} \\ &= P_a(T_k) P_{step(a,a)} \end{aligned} \quad (4.50)$$

$$\begin{aligned} P_b(2T_k) &= P_{initial} (P_{step(a,a)} P_{step(a,b)} + P_{step(a,b)} P_{step(b,b)}) \\ &= P_b(T_k) (P_{step(a,a)} + P_{step(b,b)}) \end{aligned} \quad (4.51)$$

$$\begin{aligned} P_c(2T_k) &= P_{initial} (P_{step(a,a)} P_{step(a,c)} + P_{step(a,c)} P_{step(c,c)}) \\ &= P_c(T_k) (P_{step(a,a)} + P_{step(c,c)}) \end{aligned} \quad (4.52)$$

$$\begin{aligned} P_d(2T_k) &= P_{initial} (P_{step(a,a)} P_{step(a,d)} + P_{step(a,d)} P_{step(d,d)}) \\ &= P_d(T_k) (P_{step(a,a)} + P_{step(d,d)}) \end{aligned} \quad (4.53)$$

**Table 7 All the possible 2-step paths at the end of  $2T_k$**

initial state	first step	second step	ending state
a	a-a	a-a	a
	a-a	a-b	b
	a-b	b-b	b
	a-a	a-c	c
	a-c	c-c	c
	a-a	a-d	d
	a-d	d-d	d
	a-a	a-e	
	a-e	e-e	
	a-b	b-e	e
	a-c	c-e	
	a-d	d-e	
	a-a	a-f	
	a-f	f-f	
	a-b	b-f	f
	a-c	c-f	
	a-d	d-f	

$$\begin{aligned}
 &P_e(2T_k) \\
 &= P_{initial}(P_{step(a,a)} P_{step(a,e)} + P_{step(a,e)} P_{step(e,e)} + P_{step(a,b)} P_{step(b,e)} \\
 &+ P_{step(a,c)} P_{step(c,e)} \\
 &+ P_{step(a,d)} P_{step(d,e)})
 \end{aligned} \tag{4.54}$$

$$\begin{aligned}
 &P_f(2T_k) \\
 &= P_{initial}(P_{step(a,a)} P_{step(a,f)} + P_{step(a,f)} P_{step(f,f)} + P_{step(a,b)} P_{step(b,f)} \\
 &+ P_{step(a,c)} P_{step(c,f)} \\
 &+ P_{step(a,d)} P_{step(d,f)})
 \end{aligned} \tag{4.55}$$

Thus the probability of safety at the end of  $2T_k$  is:

$$P(\text{safety at } 2T_k) = P_a(2T_k) + P_b(2T_k) + P_c(2T_k) + P_d(2T_k) \quad (4.56)$$

Generally speaking, the analysis processes mentioned above can be summarized by the Markov chain theory. A matrix containing all the information of probability map can be formed as shown in Figure 20.

$$\begin{array}{c}
 \\
 \\
 a \\
 b \\
 c \\
 d \\
 e \\
 f
 \end{array}
 \begin{array}{c}
 \\
 \\
 a \\
 b \\
 c \\
 d \\
 e \\
 f
 \end{array}
 \begin{bmatrix}
 P_{step(a,a)} & 0 & 0 & 0 & 0 & 0 \\
 P_{step(a,b)} & P_{step(b,b)} & 0 & 0 & 0 & 0 \\
 P_{step(a,c)} & P_{step(b,c)} & P_{step(c,c)} & 0 & 0 & 0 \\
 P_{step(a,d)} & P_{step(b,d)} & P_{step(c,d)} & P_{step(d,d)} & 0 & 0 \\
 P_{step(a,e)} & P_{step(b,e)} & P_{step(c,e)} & P_{step(d,e)} & P_{step(e,e)} & 0 \\
 P_{step(a,f)} & P_{step(b,f)} & P_{step(c,f)} & P_{step(d,f)} & P_{step(e,f)} & P_{step(f,f)}
 \end{bmatrix}$$

**Figure 20 Matrix form**

In this example, the Markov transition matrix  $\mathbf{P}$  has 6 rows and 6 columns. Each element  $p_{i,j}$  in  $\mathbf{P}$  represents the probability of a step from state  $i$  to state  $j$ . It is important to notice that a bar component cannot fix itself. For this reason, the probability of a step from a lower state to a higher state is zero. Assume that a bar component will not be repaired when it is not functional,

$$p_{i,j} = \begin{cases} P_{step(i,j)} & i \geq j \\ 0 & i < j \end{cases} \quad (4.57)$$

$$\mathbf{P}_M = [p_{i,j}] \quad (4.58)$$

Thus for the system in Figure 19

$$\mathbf{P}_M = \begin{bmatrix} P_{step(a,a)} & 0 & 0 & 0 & 0 & 0 \\ P_{step(a,b)} & P_{step(b,b)} & 0 & 0 & 0 & 0 \\ P_{step(a,c)} & P_{step(b,c)} & P_{step(c,c)} & 0 & 0 & 0 \\ P_{step(a,d)} & P_{step(b,d)} & P_{step(c,d)} & P_{step(d,d)} & 0 & 0 \\ P_{step(a,e)} & P_{step(b,e)} & P_{step(c,e)} & P_{step(d,e)} & P_{step(e,e)} & 0 \\ P_{step(a,f)} & P_{step(b,f)} & P_{step(c,f)} & P_{step(d,f)} & P_{step(e,f)} & P_{step(f,f)} \end{bmatrix} \quad (4.59)$$

#### 4.7 Conclusion and summary

Generally speaking, for an  $N$ -bar truss structural system, there are  $M = 2^N$  system states at a certain point in time. After a period of time, the system can move from one state to another.

The system transition matrix is defined as

$$\mathbf{P}_M = \begin{bmatrix} P_{step(a,a)} & \cdots & P_{step(n,a)} \\ \vdots & \ddots & \vdots \\ P_{step(a,n)} & \cdots & P_{step(n,n)} \end{bmatrix} \quad (4.60)$$

If the bar components are not repaired, the probability that the system move from a lower system state to a higher system state will be to zero. That is to say

$$p_{i,j} = \begin{cases} P_{step(i,j)} & i \geq j \\ 0 & i < j \end{cases} \quad (4.61)$$

Suppose that vector  $\mathbf{s}(nT_k)$  the probabilities of each system state at time  $nT_k$ ,

then initial state  $\mathbf{s}(0)$  is

$$\mathbf{s}(0) = \begin{bmatrix} p_{initial(a)} \\ p_{initial(b)} \\ p_{initial(c)} \\ \vdots \\ p_{initial(n)} \end{bmatrix} \quad (4.62)$$

where



$p_{initial}(i)$  is the probability that system is in state  $i$  initially. Notice that

$$\sum_{i=1}^M p_{initial}(i) = 1 \quad (4.63)$$

$$\sum_{i=1}^M s(nT_k)_i = 1 \quad (4.64)$$

Hence, as discussed above

$$\mathbf{s}(nT_k) = \mathbf{P}_M^n \mathbf{s}(0) \quad (4.65)$$

, and

$$\mathbf{s}((n+1)T_k) = \mathbf{P}_M \mathbf{s}(nT_k) \quad (4.66)$$

Finally, the survival probability of the system at time  $nT_k$  is

$$P_{system_s}(nT_k) = \sum_{\substack{i= \\ \text{functional states}}} s(nT_k)_i \quad (4.67)$$

The transition matrix  $\mathbf{P}_M$  has two important properties: (a) the summation of each column vector equals one, (b) every element in this matrix is smaller than one. These two properties guarantee the fact that that the  $\mathbf{P}_M$  matrix will have  $N$  eigenvalues. Among these eigenvalues, only one of them is one, and all the others are smaller than one. Assume that the eigenvalue matrix of  $\mathbf{P}_M$  is  $\mathbf{V}$ , and eigenvector matrix of  $\mathbf{P}_M$  is  $\mathbf{D}$

$$\mathbf{P}_M = \mathbf{V}^{-1} \mathbf{D} \mathbf{V} \quad (4.68)$$

, where  $\mathbf{D}$  is a diagonal matrix. On the diagonal of matrix  $\mathbf{D}$ , there is only one element that equals one (assume it is the  $k^{\text{th}}$  element), and all the other elements are zeros.

Since

$$\mathbf{P}_M^n = \mathbf{V}^{-1} \mathbf{D}^n \mathbf{V} \quad (4.69)$$

Plug equation 4.69 back to equation 4.65

$$\mathbf{s}(nT) = \mathbf{V}^{-1} \mathbf{D}^n \mathbf{V} \mathbf{s}(0) \quad (4.70)$$

$$\mathbf{V} \mathbf{s}(nT) = \mathbf{D}^n \mathbf{V} \mathbf{s}(0) \quad (4.71)$$

In matrix  $\mathbf{D}$ , when  $n \rightarrow \infty$ , the diagonal elements which are smaller than one are infinitely approaching zero, and the  $k^{\text{th}}$  diagonal element (which equal one) is approaching 1. Thus

$$\lim_{n \rightarrow \infty} \mathbf{D}^n = \mathbf{E}_n \quad (4.72)$$

where

$$\mathbf{E}_n(i, j) = \begin{cases} 1 & i = k, j = k \\ 0 & \text{all the other situations} \end{cases} \quad (4.73)$$

Plug equation 4.73 back to 4.70

$$\lim_{n \rightarrow \infty} \mathbf{D}^n = \mathbf{E}_n \quad (4.74)$$

$$\lim_{n \rightarrow \infty} \mathbf{V}^{-1} \mathbf{D}^n \mathbf{V} \mathbf{s}(0) = \mathbf{V}^{-1} \mathbf{E}_n \mathbf{V} \mathbf{s}(0) = \mathbf{s}_{st} \quad (4.75)$$

The vector  $\mathbf{s}_{st}$  is the stationary vector for matrix  $\mathbf{P}_M$ . This stationary vector not only indicates the final probabilities of each state, but reveals the performance of the associated truss as well.

## Chapter 5

### Life-cycle analysis of trusses

During its lifetime, a truss structure may experience different kinds of deterioration effects. Some of them will take place slowly and last a long period of time. As one of the long term deterioration effects, corrosion reduces the cross section area and thus makes the truss structure less reliable. Both the corrosion initiation time and the corrosion rate are uncertain. They can be modeled as random variables and can be simulated stochastically. On the other hand, maintenance can be applied to a truss structure. Both the corrosion effects and maintenance actions can be modeled stochastically. This is the main topic of this chapter.

#### 5.1 Modeling of corrosion

Corrosion may be one of most significant deterioration effects for truss structures. Corrosion always lasts a long period of time and happens randomly. A bar component suffering from corrosion will end up with reduced cross section area.

Albrecht and Naeemi [7] made a study about the corrosion speed of some steel members, and modeled the effect of corrosion by [7]

$$C(t) = At^B \quad (5.1)$$

In the equation above,  $C(t)$  is the average corrosion penetration, and  $A$  and  $B$  are two factors determined by environmental conditions.

In 2010, Frangopol and Kim developed a method to numerically simulate the corrosion [8]. In their paper, the corrosion initiation time is modeled as

$$T_{corr} = \frac{x^2}{4D \left( \operatorname{erfc}^{-1} \left( \frac{C_{th}}{C_0} \right) \right)^2} \quad (5.2)$$

Therefore, the cross section area at time  $t$  (years) is [8],

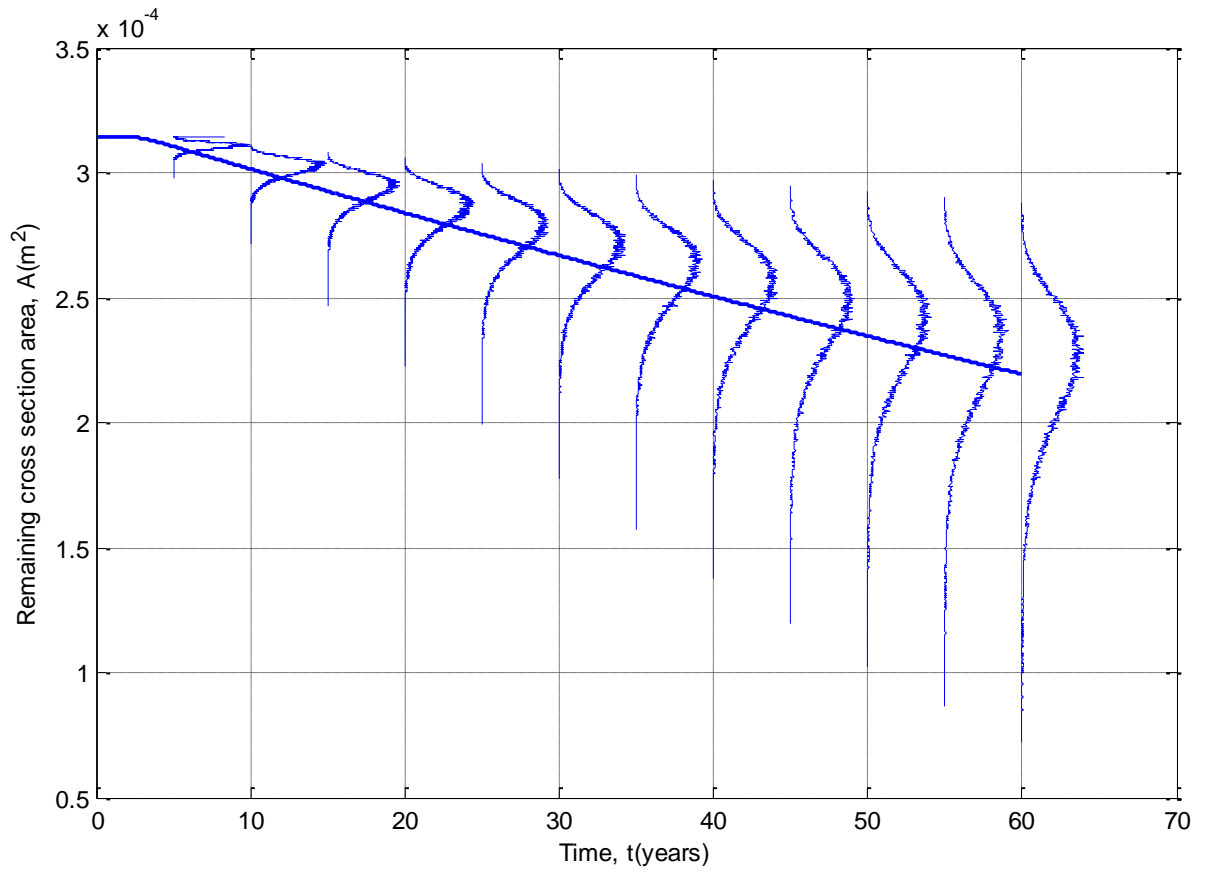
$$A_s(t) = \begin{cases} \frac{\pi d_{st0}^2}{4} \\ \frac{\pi [d_{st0} - r_{corr}(t - T_{corr})]^2}{4} \end{cases} \quad (5.3)$$

where  $d_{st0}$  is the initial diameter and  $r_{corr}$  is the corrosion rate. The simulated results of equation (5.3), based on the assumptions indicated in Table 8, are shown in Figure 21. In this study, it is assumed that, for every bar component, corrosion starts at the same time and the corrosion rate is the same.

**Table 8 Random variables characterizing corrosion**

	Variable	Mean	cov	Type of distribution
Corrosion Rate(initial corrosion)	$r_{corri}(mm/month)$	5.82E-06	0.1	Lognormal
Corrosion Initiation Time	$T_{corr}(month)$	36	0.2	Lognormal

The remaining cross sectional area of truss members under corrosion is shown in Figure 21 [8].



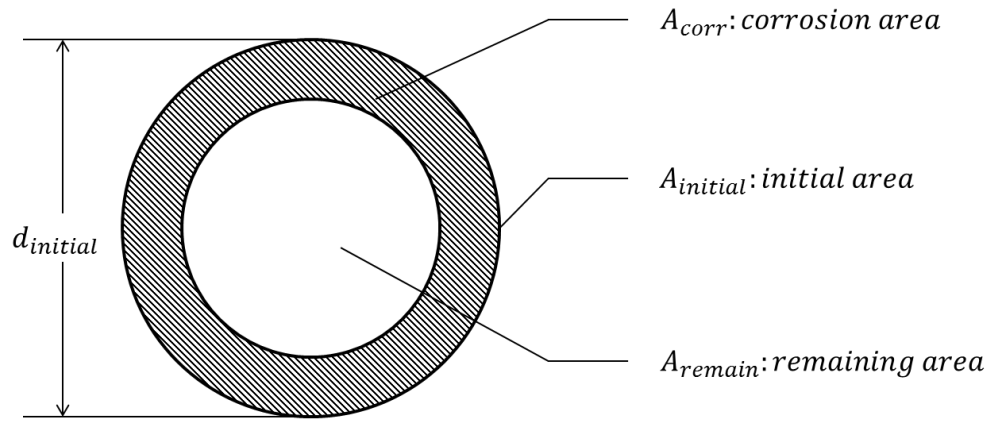
**Figure 21 Remaining cross sectional area [8]**

The line in Figure 21 represents the mean value of remaining cross section area at the end of each time period, and the vertical curves represent the PDFs of the remaining cross area. The area of cross section is decreasing while its dispersion is increasing.

The ratio of reduction of cross section area is defined as

$$\rho_A = \frac{A_{corr}}{A_{initial}} \quad (5.4)$$

where  $A_{corr}$  is the corrosion area and  $A_{initial}$  is the initial area as shown in Figure 22.



**Figure 22 Corrosion of cross section**

Corrosion of a truss structure can be classified into six stages. They are defined in the table below.

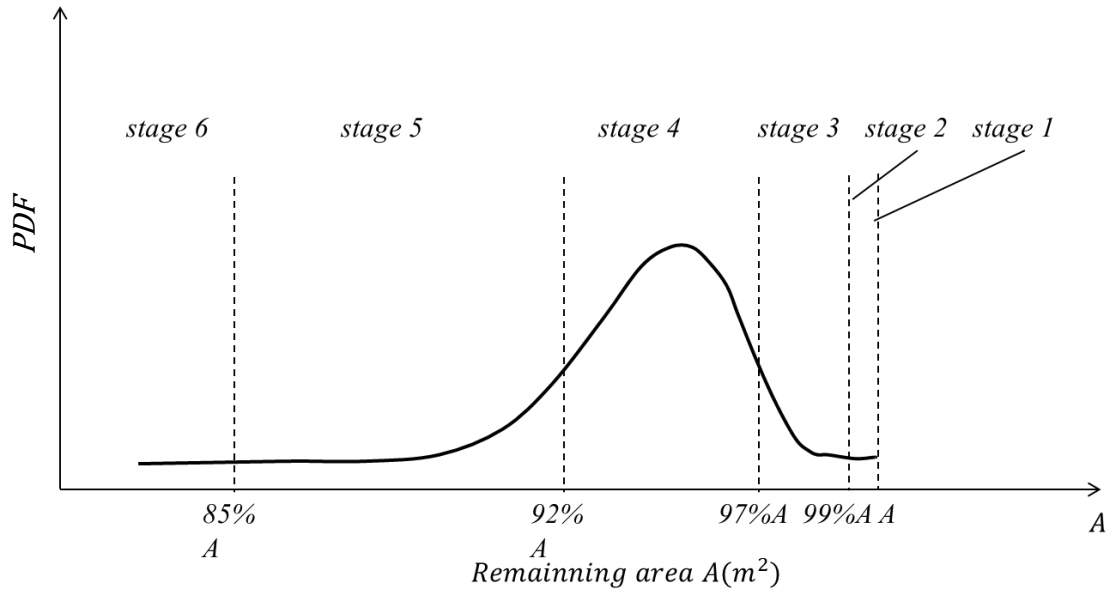
**Table 9 Classification of corrosion**

Corrosion stages	Condition	Corrosion ratio
stage 1	Nonecorrosion	0
stage 2	Slight corrosion	( 0 , 1% ]
stage 3	Moderate Corrosion	( 1% , 3% ]
stage 4	Heavy Corrosion	(3% , 8% ]
stage 5	Severe Corrosion	(8% , 15% ]
stage 6	Not functional	>15%

The probability that system moves from stage  $i$  corrosion to stage  $j$  corrosion over a time period  $T_k$  is defined as

$$p_{corr(i,j)} = \text{probability to move from stage } i \text{ to stage } j \quad (5.5)$$

This probability can be quantified by the method indicated in [8]. For example, the system is at stage 1, and then the histogram of remaining area after a time period  $T_k$  can be simulated as indicated below.



**Figure 23 Histogram of remaining area after a time period of  $T_k$**

The associated probabilities to move from the one corrosion stage to another corrosion stage can be simulated. It is important to notice that the probability to move from corrosion stage 1 to any other stage of corrosion varies with time because of initiation time of corrosion. Generally, the associated time of corrosion for  $p_{corr(1,j)}(nT_k)$  is

$$T_{corr} - (n - 1)T_k \quad (5.6)$$

The probabilities to move from one corrosion stage to another one can be written as a matrix

$$\begin{aligned}
 & \mathbf{P}_{corr}(nT_k) \\
 &= \begin{bmatrix} p_{corr(1,1)}(nT_k) & 0 & 0 & 0 & 0 & 0 \\ p_{corr(1,2)}(nT_k) & p_{corr(2,2)} & 0 & 0 & 0 & 0 \\ p_{corr(1,3)}(nT_k) & p_{corr(2,3)} & p_{corr(3,2)} & 0 & 0 & 0 \\ p_{corr(1,4)}(nT_k) & p_{corr(2,4)} & p_{corr(3,3)} & p_{corr(4,4)} & 0 & 0 \\ p_{corr(1,5)}(nT_k) & p_{corr(2,5)} & p_{corr(3,4)} & p_{corr(4,5)} & p_{corr(5,5)} & 0 \\ p_{corr(1,6)}(nT_k) & p_{corr(2,6)} & p_{corr(3,5)} & p_{corr(4,6)} & p_{corr(5,6)} & p_{corr(6,6)} \end{bmatrix} \quad (5.7)
 \end{aligned}$$

$P_{corr}(nT_k)$  can be simulated by the method discussed above. The parameters of this simulation are listed in Table 10. Here we assume that the time period equals one month ( $T_k = 1$  month) and  $n = 0,1,2,3, \dots,720$  (60 years).

**Table 10 Random variables**

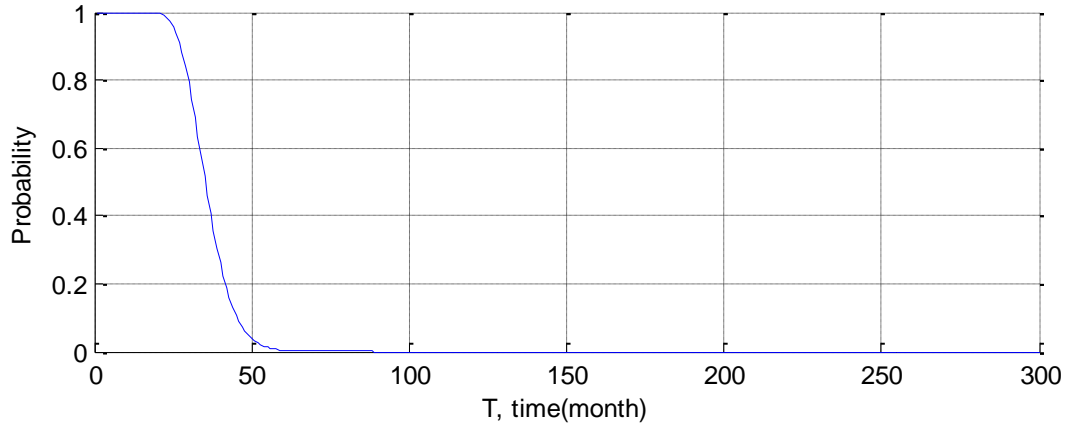
	Variables	Mean	cov	Type of distribution
Corrosion Rate(initial corrosion)	$r_{corri}(mm/year)$	5.82E-06	0.1	Lognormal
Corrosion Rate	$r_{corr}(mm/year)$	1.16E-05	0.1	Lognormal
Corrosion Initiation Time	$T_{corr}(month)$	36	0.2	Lognormal

Figure 24 shows the probabilities to move from corrosion stage 1 to all the other corrosion stages. Figure 25 shows the histogram of remaining cross section area at the end of year 5, year 10, year 30, and year 50. Notice that

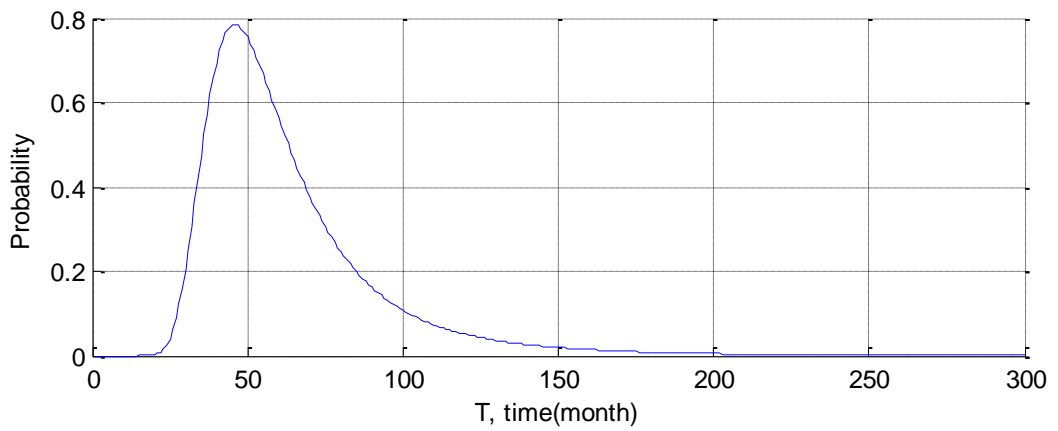
$$\sum_{for\ all\ \beta} p_{corro(\alpha,\beta)} = 1 \quad (5.8)$$



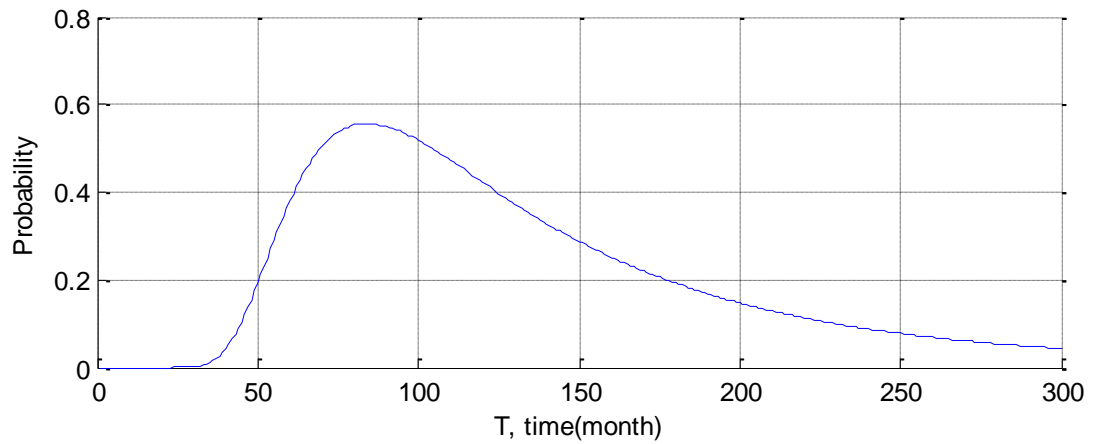
Probability to remain in stage 1

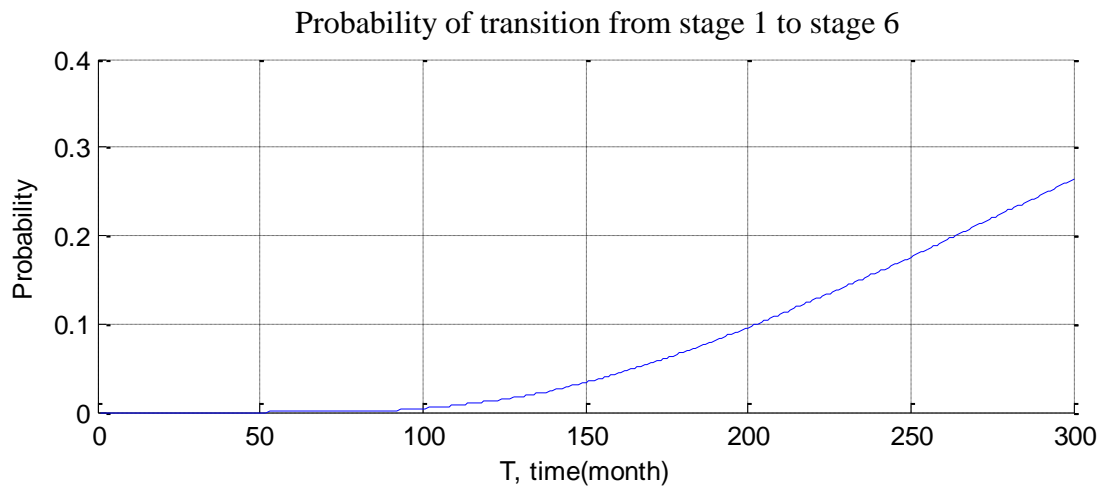
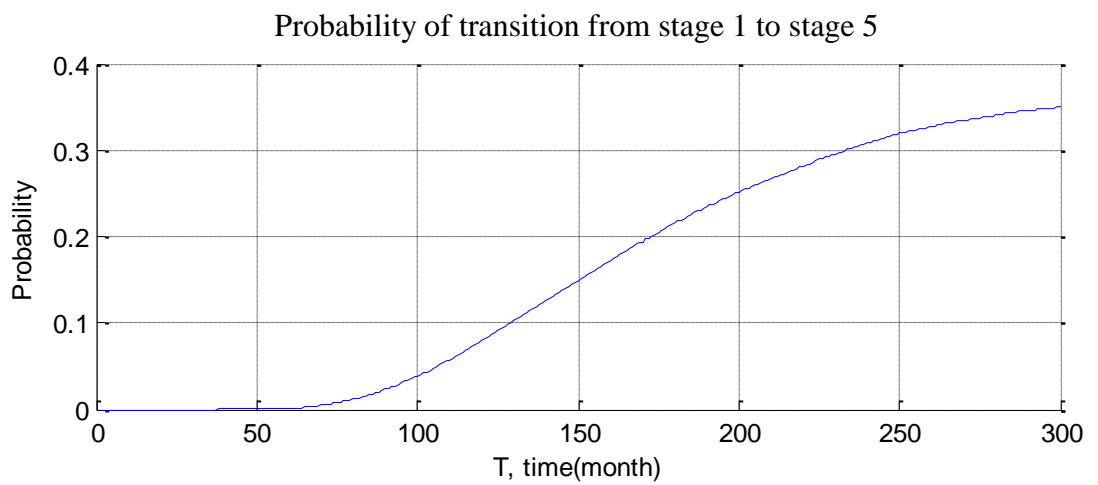
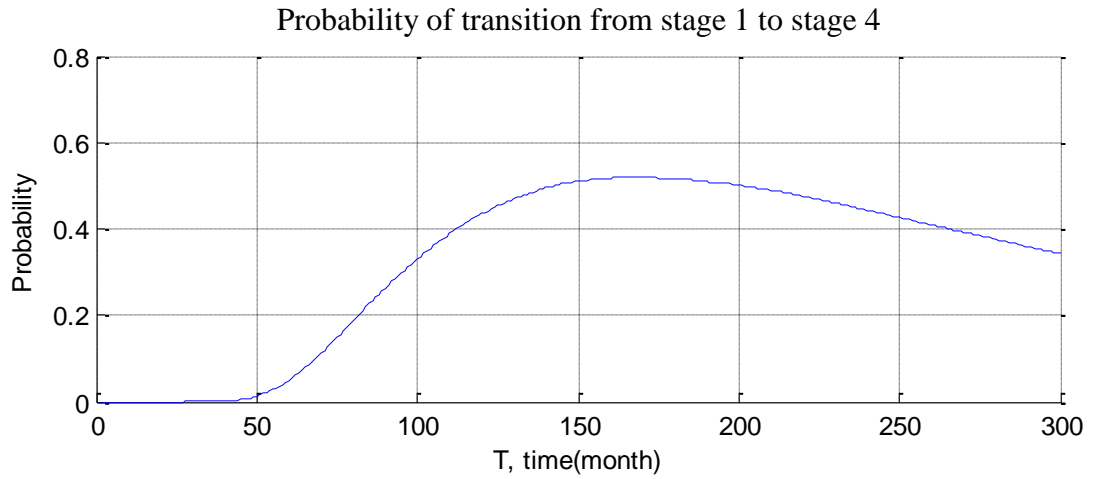


Probability of transition from stage 1 to stage 2

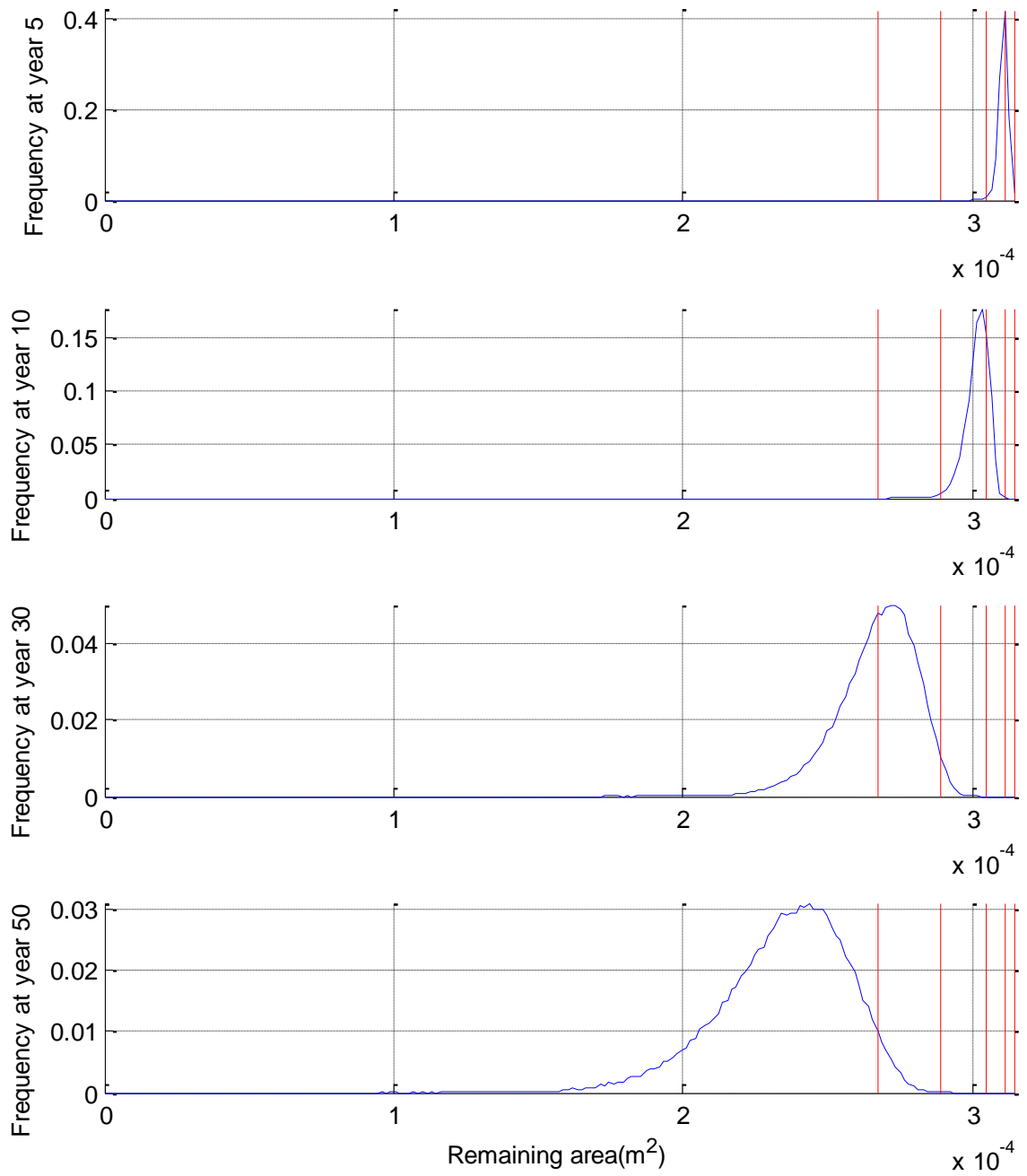


Probability of transition from stage 1 to stage 3

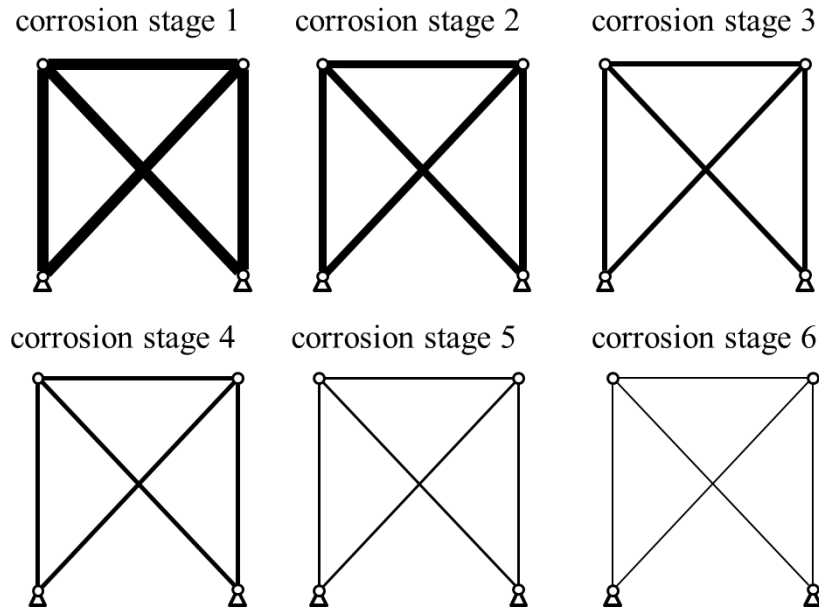




**Figure 24 Probabilities of transition from stage 1 to other stages**



**Figure 25 Histogram of remaining cross section area at the end of year 5, 10, 30 and 50**



**Figure 26 The same structural state at different corrosion stages**

As shown in Figure 26, a truss can be at different corrosion stages, so a system state should be represented by two numbers as

$$state(i, k) \quad (5.9)$$

Here  $i$  represents the structural state, and  $k$  represents the corrosion stage. The associated probability that the system is in one corrosion condition is calculated as

$$\begin{aligned} &P(\text{system at state } (32,2)) \\ &= P(\text{structure state} = 32)P(\text{corrosion state} = 2) \quad (5.10) \end{aligned}$$

Generally speaking, at a certain point in time, for a truss structural system with  $N$  components, there are  $2^N$  structural states and 6 corrosion stages. Therefore, there are  $6 \times 2^N$  different possible system states when corrosion effect is considered. The new system state vector is defined as

$$s_c(nT_k) = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \end{bmatrix} \quad (5.11)$$

Here,  $s_k$  represents the probabilities of a certain structural state associated with corrosion state  $k$ .

$$s_k = \begin{bmatrix} P(\text{state}(1, k)) \\ P(\text{state}(2, k)) \\ P(\text{state}(3, k)) \\ \vdots \\ P(\text{state}(n, k)) \end{bmatrix} \quad (5.12)$$

Let  $P_{sk}$  represents transition matrix for system in corrosion stage  $k$

$$P_{sk} = \begin{bmatrix} p_{k1,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ p_{k1,N} & \cdots & p_{kN,N} \end{bmatrix} \quad (5.13)$$

where

$$p_{kij} = \begin{cases} p_{k(i,j)} & i \geq j \\ 0 & i < j \end{cases} \quad (5.14)$$

Therefore, the overall Markov matrix should be

$$P_c(nT_k) = \begin{bmatrix} P_{c1,1}(nT_k) & 0 & 0 & 0 & 0 & 0 \\ P_{c1,2}(nT_k) & P_{c2,2} & 0 & 0 & 0 & 0 \\ P_{c1,3}(nT_k) & P_{c2,3} & P_{c3,3} & 0 & 0 & 0 \\ P_{c1,4}(nT_k) & P_{c2,4} & P_{c3,4} & P_{c4,4} & 0 & 0 \\ P_{c1,5}(nT_k) & P_{c2,5} & P_{c3,5} & P_{c4,5} & P_{c5,5} & 0 \\ P_{c1,6}(nT_k) & P_{c2,6} & P_{c3,6} & P_{c4,6} & P_{c5,6} & P_{c6,6} \end{bmatrix} \quad (5.15)$$

where

$$P_{ci,j} = p_{corr(i,j)} P_{si} \quad (5.16)$$

Thus,

$$s_c(nT_k) = \left( \prod_{i=1}^n P_c(iT_k) \right) s_c(0) \quad (5.17)$$

The system probability of safety over a certain period of time  $T_k$  is

$$P_{sys}(nT_k) = \sum_{\substack{i=functional \\ states}} s_c(nT_k)_i \quad (5.18)$$

where  $s_c(nT_k)_i$  is the  $i^{th}$  element in  $s_c(nT_k)$ .

## 5.2 Modeling of maintenance

There are two types of maintenance. The first type of maintenance replaces the deteriorated components with new ones. The benefit of this type of maintenance is that it will improve system reliability significantly. The total cost of this type of maintenance is high. The main idea of the second type of maintenance is to prevent the truss structure from further deterioration. For example, painting or coating bar components can prevent truss structure from corrosion. Although this type of maintenance will not improve the system reliability, it can slow down the decrease of system reliability.

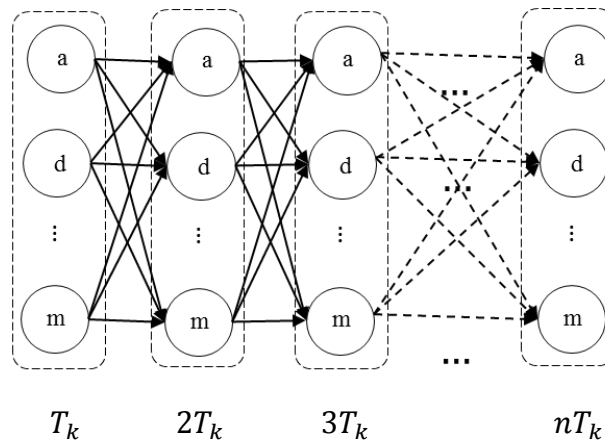
Recall the overall transition matrix without maintenance is

$$P_c(nT) = \begin{bmatrix} P_{c1,1}(nT) & 0 & 0 & 0 & 0 & 0 \\ P_{c1,2}(nT) & P_{c2,2} & 0 & 0 & 0 & 0 \\ P_{c1,3}(nT) & P_{c2,3} & P_{c3,3} & 0 & 0 & 0 \\ P_{c1,4}(nT) & P_{c2,4} & P_{c3,4} & P_{c4,4} & 0 & 0 \\ P_{c1,5}(nT) & P_{c2,5} & P_{c3,5} & P_{c4,5} & P_{c5,5} & 0 \\ P_{c1,6}(nT) & P_{c2,6} & P_{c3,6} & P_{c4,6} & P_{c5,6} & P_{c6,6} \end{bmatrix} \quad (5.19)$$

Thus the transition matrix containing the probabilities of maintenance is

$$P_{cm}(nT) = \begin{bmatrix} P_{c1,1}(nT) & P_{m2,1} & P_{m3,1} & P_{m4,1} & P_{m5,1} & P_{m6,1} \\ P_{c1,2}(nT) & P_{c2,2} & P_{m3,2} & P_{m4,2} & P_{m5,2} & P_{m6,2} \\ P_{c1,3}(nT) & P_{c2,3} & P_{c3,3} & P_{m4,3} & P_{m5,3} & P_{m6,3} \\ P_{c1,4}(nT) & P_{c2,4} & P_{c3,4} & P_{c4,4} & P_{m5,4} & P_{m6,4} \\ P_{c1,5}(nT) & P_{c2,5} & P_{c3,5} & P_{c4,5} & P_{c5,5} & P_{m6,5} \\ P_{c1,6}(nT) & P_{c2,6} & P_{c3,6} & P_{c4,6} & P_{c5,6} & P_{c6,6} \end{bmatrix} \quad (5.20)$$

### 5.3 Performance analysis and conclusion



**Figure 27 System states at several points in time**

Figure 27 illustrates a network of all system states at various points in time. In this network, each node stands for a system state at a certain point in time. The nodes are classified into different columns as shown in the figure, and each column stands for a time point. The connection between two nodes represents the probability of transition from one state to another.

The most important information contained in this graph is the system reliability at each time point. At the time  $nT_k$ , the truss structural system may be in any system state. Some of the system states are functional states, and others are failed states. Both corrosion and maintenance can be incorporated using the proposed approach.

Each path in this probability map has a corresponding length. In the most probable system path, some steps may represent events of failure, and some steps may describe the events of corrosion or events of maintenance. This kind of probability map not only indicates the performance of a truss structure at a certain point in time, but reveals the evolution of a truss structure during its lifetime.



## Chapter 6

### Case study

The first part of this chapter will be an introduction to TRAP (Truss Reliability Analysis Program) and the second part of this chapter presents two case studies of two truss structures.

#### 6.1 Introduction to Truss Reliability Analysis Program

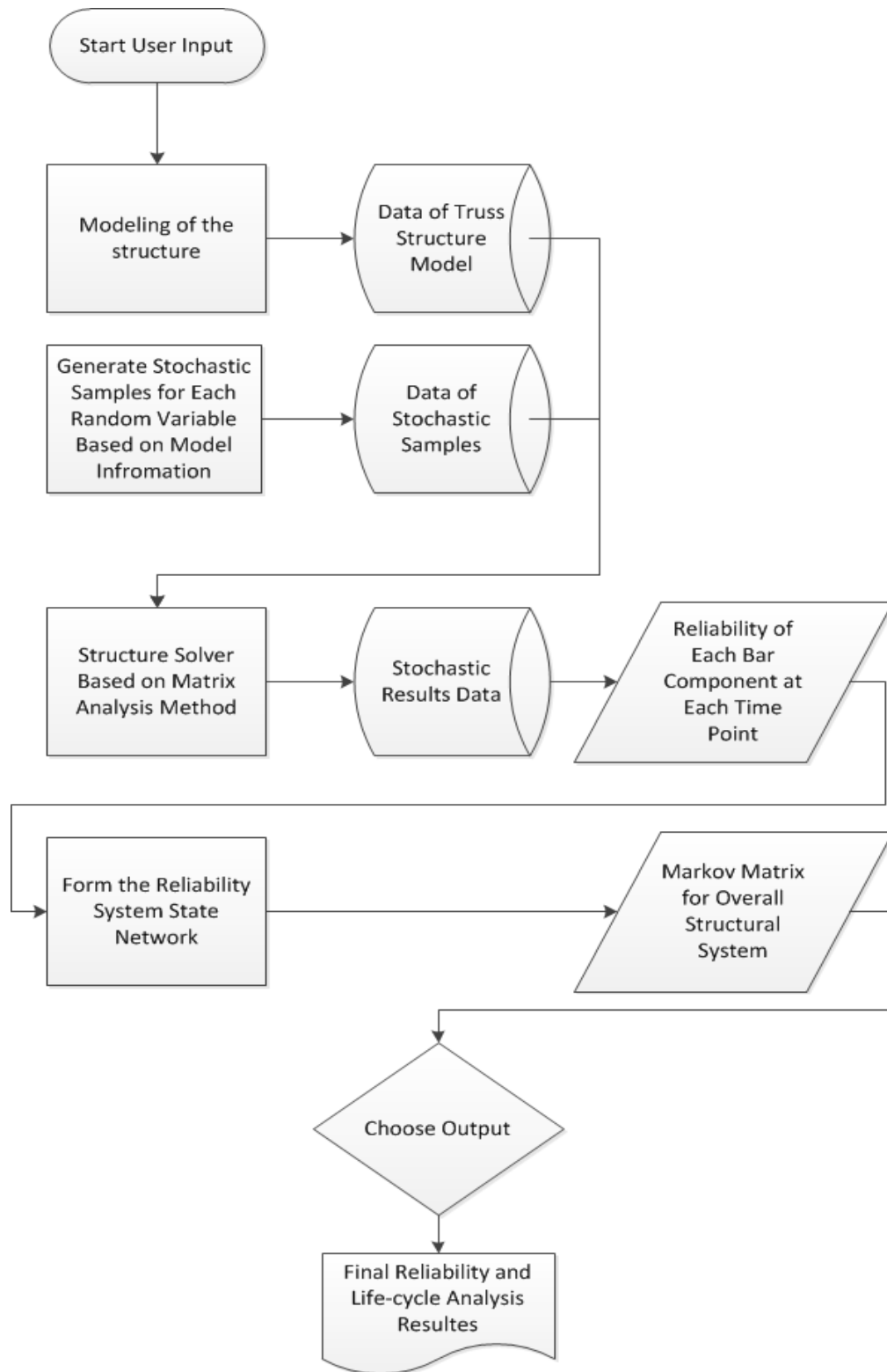
The Truss Reliability Analysis Program (TRAP) is a general purpose reliability analysis computer program developed under Matlab environment. The basic purpose of this program is to quantify the time-variant reliability for any 2D truss structure. The algorithms that TRAP employed are discussed in the previous chapters.

With all the parameters of a 2D truss structure, TRAP is able to provide the system reliability at each point in time. At each point in time, it can also provide the probabilities of each system state as well. Deterioration effects are modeled in the system as well.

Figure 28 shows the flowchart of TRAP. Firstly, a graphical user interface (GUI) shown in Figure 29 allows user to model the truss structure. This user interface is designed to collect data about the truss structure, such as material properties, cross section areas and external loads. It collects mean value and standard deviation of each random variable.

With all the parameters collected, the program will generate samples of each random variable. These samples are feed into the finite element solver for displacements at each node. Once all the displacements have been generated, the

program will simulate the failure probability of each bar component by the Monte Carlo simulation method. The transition matrix for Markov chain is then formed with the information of component reliabilities. The initial system is considered to be fully functional, and then the state distribution at each point in time is calculated. Finally, the time-variant system reliability is computed.



**Figure 28 Flowchart of TRAP**

Figure 30 shows the dialog box for node input. X and Y are the two coordinates of a node.

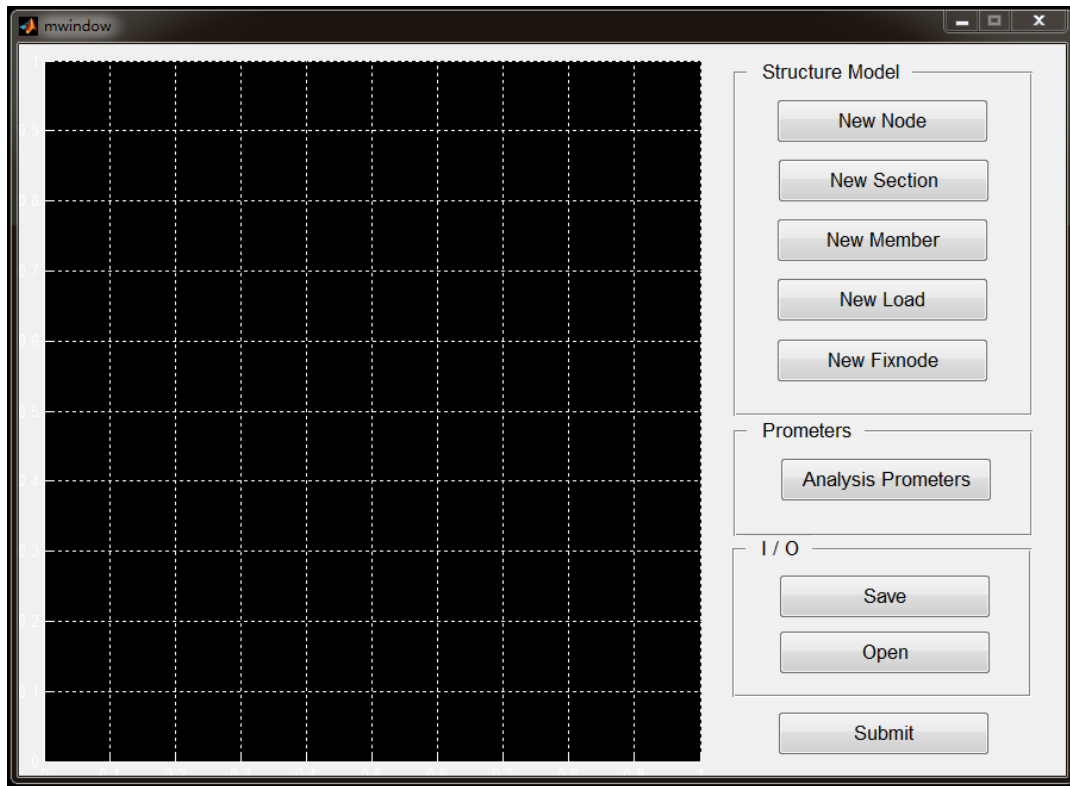


Figure 29 Main Graphic User Interface window

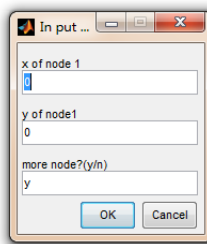
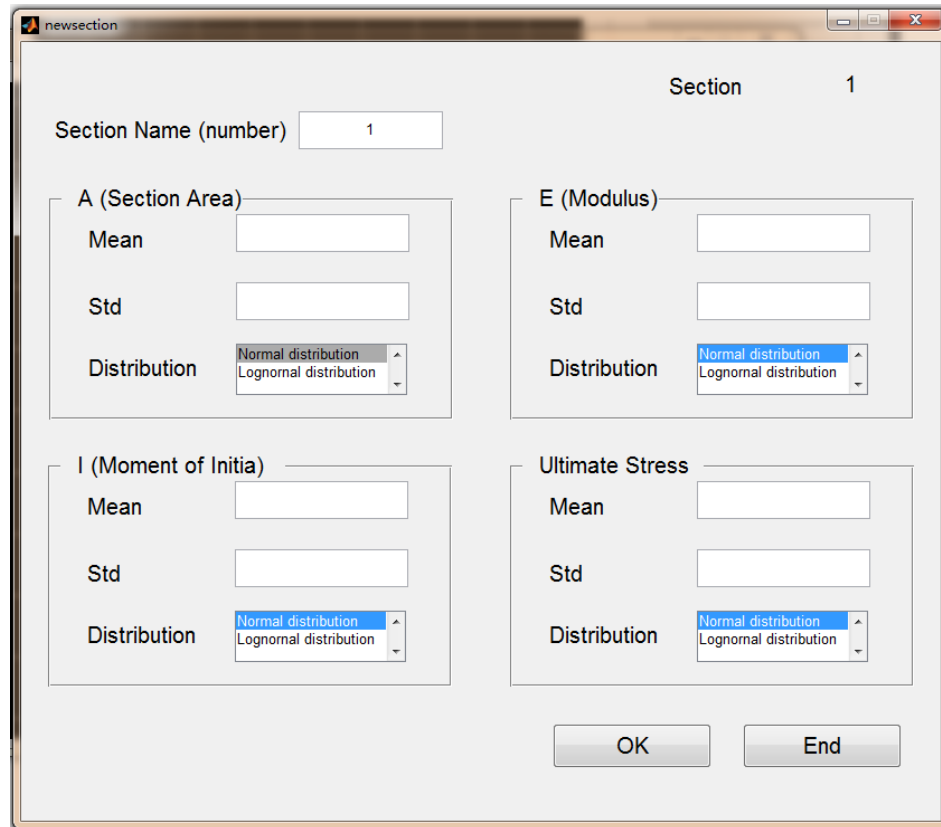
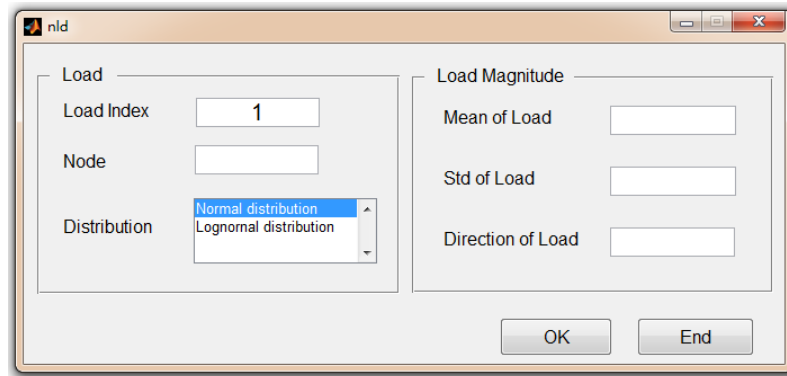


Figure 30 Input nodes



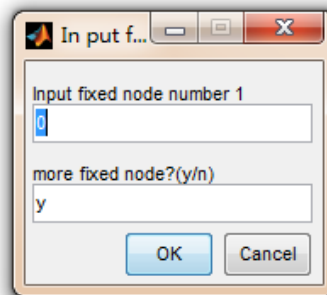
**Figure 31 Input section properties**

Figure 31 demonstrates the input dialog box for section properties. There are several options in this box, which are separated in different groups. Group “A”, group “E”, group “I” group “Ultimate Stress” each stands for cross section area, Young’s modulus, moment of inertia and ultimate stress of this type of bar component. In each group, user can input mean value (Mean), standard deviation (Std) and choose one of the two types of distribution.



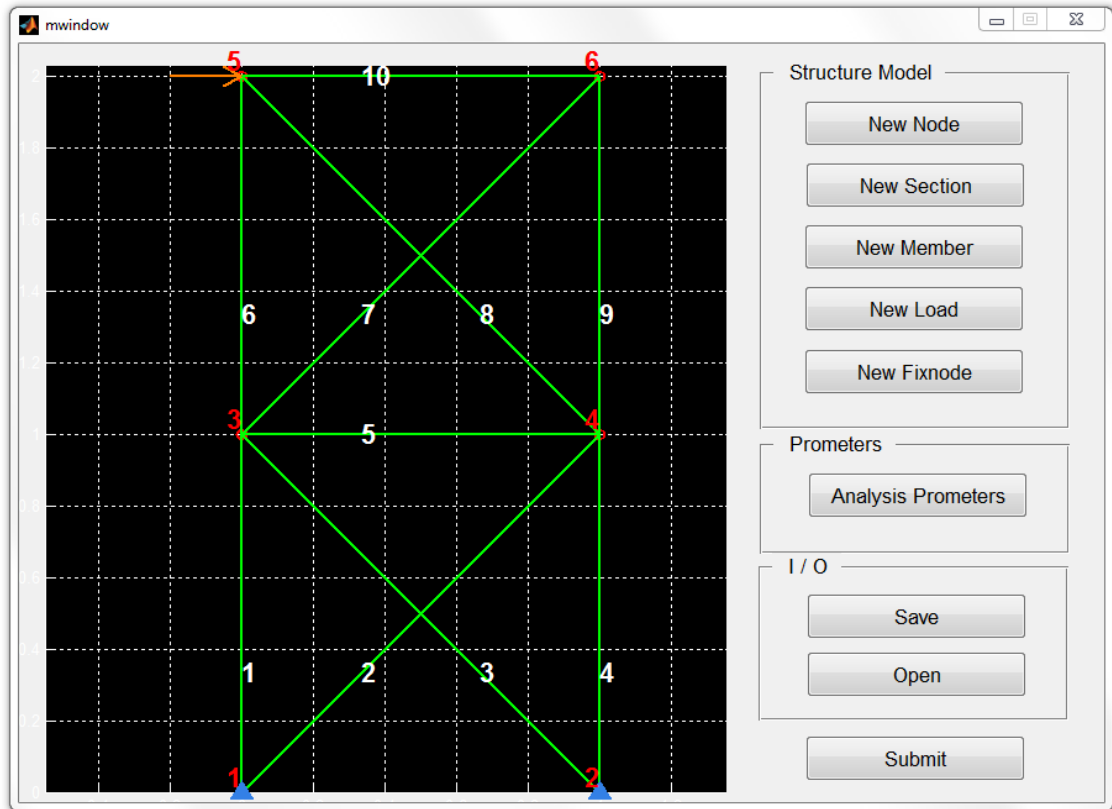
**Figure 32 Input of Load**

Figure 32 Input of Load shows the dialog box of load input. The “Node” input stands for the node on which acts the load. Notice that the direction of the load is in radians.



**Figure 33 Input of boundary condition**

Figure 33 Input of boundary condition shows the dialog box of boundary condition input. The boundary nodes are fixed in both x and y directions.



**Figure 34 Modeling of a ten bar truss structure**

Finally, Figure 34 shows the modeling of a 10-bar truss structure.

TRAP is written as a collection of functions. The descriptions and datalists are listed below.

## 1. Modeling

- **mwindow.m**

*The mwindow.m defines a graphical user interface that is designed for user to build truss structure model. The data will be saved as below:*

- **node**

node: [(x) (y)]  
 noden: (index of node)

- **section**

section:

[(section name)

(mean of section A) (std of section A) (distribution)

(mean of section E) (std of section E) (distribution)

(mean of section I) (std of section I) (distribution)

(mean of section sigm) (std of section sigm) (distribution)

]

sectionn: (number of sections)

- **fix node**

fx: (node)

fxn: (index of fx)

- **load**

ld: [(node) (mean) (std) (distribution) (direction)]

ldn: (index of member)

- **member**

mem: [(node1) (node2) (section index)]

memn: (number of members)

- **samplen (total number of samples)**

samplen: (number of samples)

- **Ecovp (correlation of E)**

Ecovp: (cov of E each member)

- **Acovp (correlation of A)**

Acovp: (cov of A each member)

- **Sgcovp (correlation of S)**

Sgcovp: (cov of Sg each member)

“save” will save model data to ‘.../data/inputdata.mat’. Structure of inputdata.mat is



.node  
.noden  
.section  
.sectionn  
.mem  
.memn  
.ld  
.ldn  
.fx  
.fxn  
.samplen  
.Ecovp  
.Acovp  
.Sgcovp

## 2. sampling

---

- **gnrtsample.m**

*Sample generator is a function that will generate all the samples and save them in folder of sample data.*

Input:

discount ---- reduction of EA

fdiscout ---- reduction of external loading force

Output:

Esample ---- sample of E

Asample ---- sample of A

Fsample ---- sample of F

Sgsample ---- sample of ultimate stress

Save:

.EAsample

. Fsample  
. Sgsample

Save As:

\data\sampladata'dis''fdis'.mat

### 3. solving

---

- **gstiff.m**

*This function returns stiffness matrix K of choosing system state, with input E and A of each member.*

Input:

E ---- line vector, E of each member

A ---- line vector, A of each member

sst ---- line vector, represents system state.(If a certain member is functioning, it will be represented as "1", otherwise "0");]

Output:

K ---- stiffness matrix of a certain state

- **bdstiff**

*This function will impose boundary condition to input matrix.*

Input:

Ko ----- input matrix

Output:

K ---- matrix with boundary condition

- **gsst.m**

*This function will generate all the system states according to total number of members. Also, it will generate the states that are not singular and save them in sstdata.nsst. The fsst and the failure modes will also be saved.*

Input:

Output:

Save:

.sst ---- all the system state

.sigu ---- if a certain sst is singular or not  
.nsst ---- all the modes that are non-singular  
.fsst ---- all the failure modes developed from nsst

Save as:

'...\data\sstdata.mat', 'sst', 'sigu'

- **stress.m**

*With results of displacements at each DOF, it will return stress in each member.*

Input:

ssti ---- state of system, live vector

u ---- displacement at each DOF

Output:

stressn ---- stress in each member, line vector

Save:

nan

- **pm.m**

*This function finds structural reliability by Monte Carlo method.*

Input:

Output:

Save:

.K ---- stiffness of each sample, three dimensional matrix (m,n,sn)

.strs ---- stress in each member of each sample (sn,stresses)

.ff ---- fail or not state of structure in each sample (sn,state)

.pfail ---- probability of failure of each member(1,p of each member)

Save as:

'.../output/pmdata.mat'

- **importancei.m**

*This function calculate importance factor of member i as defined by Leemis Definition 2.7*

Input:

Output:

Save:

.imp ---- importance factor of each member

Save as:

‘/output/impidata.mat’, ‘imp’

- **bghall.m**

This function will build system state graph.

Save:

.A ---- graph matrix  
.cA ---- connection between each states, under asst order  
.asst ---- all the state, nsst and fsst together  
.nfsst ---- number of failure states  
.allsstxy ---- x and y of each state under asst order

Save as:

‘/output/bghdata.mat’, ‘A’

- **sti.m**

*This function generates the original stiffness matrix for each member*

Save:

.koi ----- k of each member with no boundary DOF

Save as:

‘/data/stidata.mat’

- **meank.m**

*This function will form the mean value stiffness matrix and its inverse of each nonsingular system state.*

Input:

eE ----- mean of E of each member, line vector  
eA ----- mean of A of each member, line vector

Output:

mK ---- stiffness matrix of each state, 3D, (:,:,i);  
imK ---- inverse of mK, 3D, (:,:,i);  
mKindex ---- system index of each mK, line vector;

- **gu.m**

*This function returns mean value, cov and var of displacement at each DOF*

Input:

discount ----- reduction of EA  
f ----- external force

ssti ----- certain system state

Output:

eu ----- mean of displacement at each DOF

covu ----- cov of displacement at each DOF

varu ----- var of displacement at each DOF

- **gforce.m**

*This function returns external loading force at each DOF with boundary condition*

Input:

discount ----- reduction of f

Output:

df ----- loading force at each DOF

- **gpf.m**

*This function returns probability of failure of each member of a certain state at certain kind of section*

Input:

discount ----- reduction of section properties

fdiscout ----- reduction of external force

ssti ----- system state

Output:

mempf ----- probability of failure of each member, if member not exist, return nan

- **grf.m**

*This function generates and save mean, std and type of distribution of nodal force at each node.*

Input:

Output:

Save:

uf ----- mean value of f, matrix, each column represent nodal force by an external force

stdf ----- std of nodal forces, (2noden x ldn) matrix

tf ----- type of distribution of f, line vector

Save as:

‘/data/grfdata.mat’

- **lnag.m**

*This function generates length and angle of each member and save them.*

INOUT:

OUTPUT:

SAVE:

memlen ----- length of each member

memang ----- angle of each member

‘/data/lnagdata.mat’

- **gpA.m**

*This function generates length and angle of each member and save them.*

Input:

discount ----- reduction of of EA

fdiscout ---- reduction of of force

Output:

pA ---- probability transformation matrix

Save:

memlen ----- length of each member

memang ----- angle of each member

Save as:

‘/data/lnagdata.mat’

---

---

All the data is save in .mat files. The structures of each .mat file are listed below.

---

---

## **model data**

---

- **inputdata.mat**

*model information saved as:*

.node

.noden

.section

.sectionn

. mem

. memn

. ld

.ldn  
.fx  
.fxn  
.samplen  
.memcov

- **inputdatao.mat**

*copy of inputdata.mat*

- **inputdata\_t.f**

*Model information saved as .txt file*

- **sampldata.mat**

*Generated samples saved as:*

.A : sn x member  
.E : sn x membern  
.force : sn x forcen  
.sigma : sn x membern

- **sstdata.mat**

*All the system state and singularity of each state saved as:*

.sst ---- all the system state  
.sigu ---- if a certain sst is singular or not  
.nsst ---- all the modes that are non singular  
.fsst ---- all the failure modes developed from nsst

- **stidata.mat**

All original matrix of each member

.koi ----- original k of each member

## output data

---

- **pmdata.mat**

Results generated by Monte Carlo simulation saved as:

- .K ---- stiffness of each sample, three dimensional matrix (m,n,sn)
- .strs ---- stress in each member of each sample (sn,stresses)
- .ff ---- fail or not state of structure in each sample (sn,state)
- .pfail ---- probability of failure of each member(1,p of each member)

- **impidata.mat**

Importance factor of each member saved as:

- .imp ---- importance factor of each member

- **bgh.mat**

system failure model graph

- .A ---- graph matrix
  - .A ---- graph matrix
  - .cA ---- connection between states, under asst order
  - .asst ---- all the state, nsst and fsst together
  - .nfsst ---- number of failure states
  - .allsstxy ---- x and y of each state under asst order
- 
- 

## 6.2 Case study 1: analysis of a ten-bar truss structure without corrosion

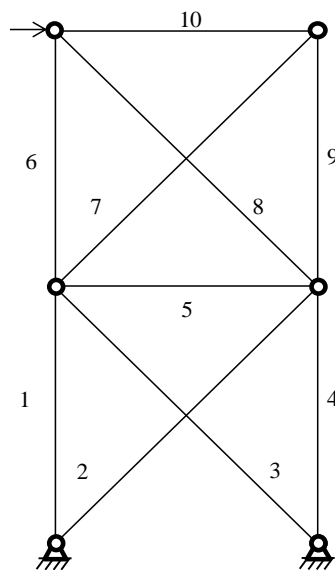


Figure 35 10-bar truss structure



**Table 11 Parameters of the 10 bar truss structure**

Item	Mean	COV	Type of Distribution
A1 (square meter)	4.80E-04	1.00E-01	Lognormal
A2 (square meter)	4.80E-04	1.00E-01	Lognormal
A3 (square meter)	4.80E-04	1.00E-01	Lognormal
A4 (square meter)	4.80E-04	1.00E-01	Lognormal
A5 (square meter)	3.20E-04	1.00E-01	Lognormal
A6 (square meter)	3.20E-04	1.00E-01	Lognormal
A7 (square meter)	4.80E-04	1.00E-01	Lognormal
A8 (square meter)	4.80E-04	1.00E-01	Lognormal
A9 (square meter)	3.20E-04	1.00E-01	Lognormal
A10 (square meter)	3.20E-04	1.00E-01	Lognormal
E (Pa)	2.00E+09	1.00E-02	Lognormal
Ultimate Stress (Pa)	2.35E+08	1.00E-02	Lognormal
Load (N)	5.30E+04	1.00E-01	Lognormal

Parameters of the ten-bar component truss structure are listed in Table 11. All random variables are considered to follow the lognormal distribution. There are two kinds of cross section. The component reliability is calculated by TRAP, and the results are listed in Table 12 and Table 13. Assume the time period is one month ( $T_k = 1$  month) and the ten-bar truss structure is at system 1024 initially. The “NAN” terms in Table 12 and Table 13 means that the corresponding bar component has already failed at that point in time.

Figure 36 Probabilities of each system state at the end of each time period shows the probabilities of each system state at each point in time. The diameter of each circle equals the probability that system is at that state. The solid circles represent the functional states and the hollow circles represent the failed system states. The Y-axis indicates system states, from state 1 to state 1024 in this case. X-axis in Figure 36 represents the time. The time increment is considered to be 1 month ( $T_k = 1$  month). Structural state 1024, state 960, state 832, state 512, state 448, state 416,

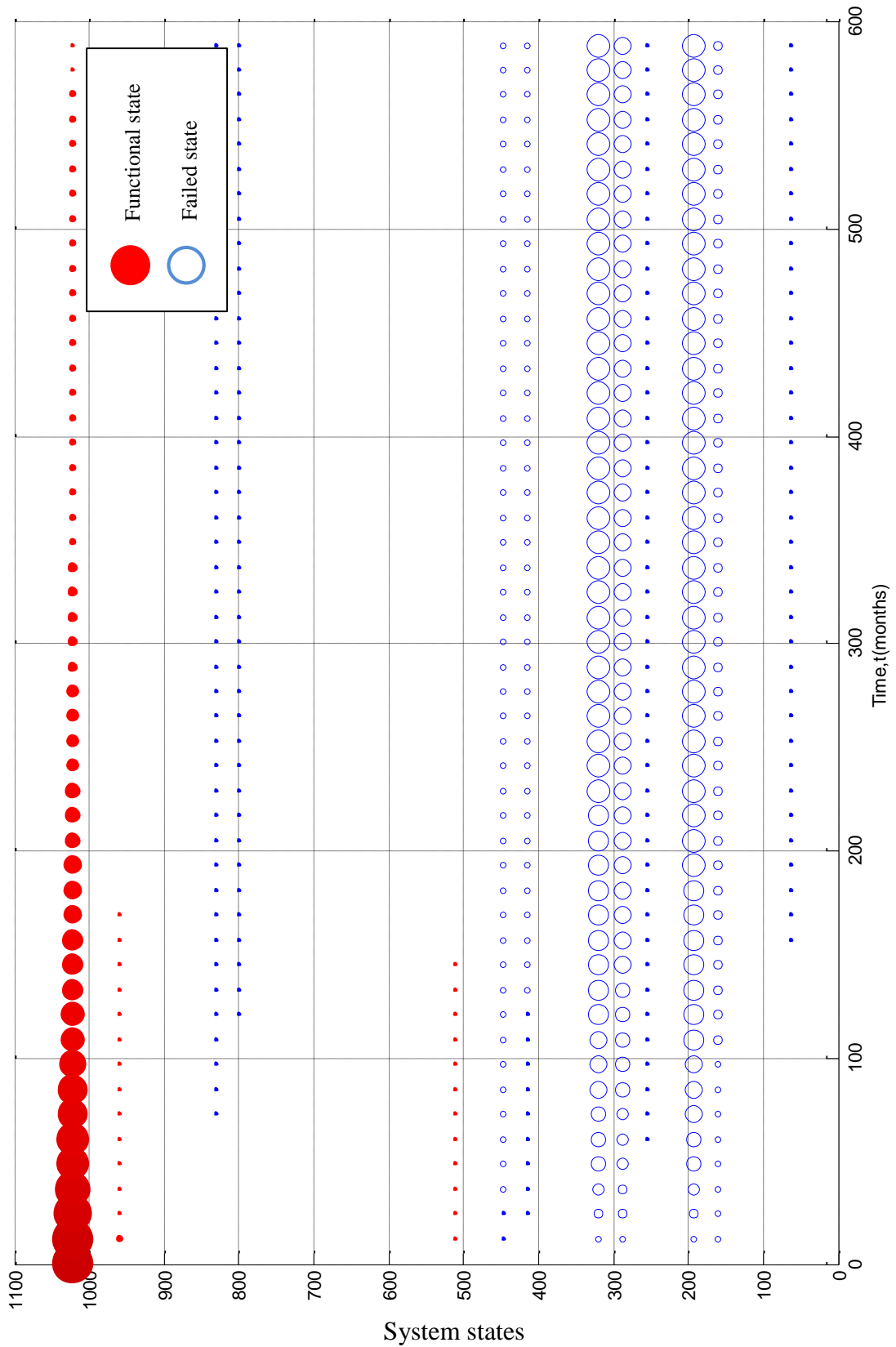
state 320, state 288, state 192 and state 160 are the most significant states. Table 14 shows the most significant states and the associated probabilities at the end of each 5 years.

**Table 12 Component reliability in each system state (a)**

Bar component system state	1	2	3	4	5	6	7	8	9	10
1024	0.0041	0.0000	0.0000	0.0053	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1023	0.0013	0.0000	0.0000	0.0134	0.0000	0.0064	0.0000	0.0021	0.0000	NaN
1022	0.0014	0.0000	0.0000	0.0138	0.0000	0.0068	0.0000	0.0018	NaN	0.0000
1020	0.0143	0.0000	0.0000	0.0014	0.0000	0.0000	0.0022	NaN	0.0068	0.0068
1016	0.0012	0.0000	0.0000	0.0139	0.0000	0.0063	NaN	0.0017	0.0000	0.0000
1008	0.0130	0.0000	0.0000	0.0013	0.0000	NaN	0.0018	0.0000	0.0066	0.0066
992	0.0029	0.0000	0.0000	0.0100	NaN	0.0000	0.0000	0.0000	0.0000	0.0000
991	0.0000	0.0018	0.0000	0.3291	NaN	0.0067	0.0000	0.0018	0.0000	NaN
990	0.0000	0.0017	0.0000	0.3292	NaN	0.0071	0.0000	0.0018	NaN	0.0000
988	0.3309	0.0000	0.0018	0.0000	NaN	0.0000	0.0018	NaN	0.0068	0.0067
984	0.0000	0.0019	0.0000	0.3305	NaN	0.0070	NaN	0.0020	0.0000	0.0000
976	0.3310	0.0000	0.0019	0.0000	NaN	NaN	0.0019	0.0000	0.0066	0.0072
960	0.9929	0.0021	0.9774	NaN	0.4007	0.0000	0.0000	0.0000	0.0000	0.0000
959	0.9924	0.0018	0.9777	NaN	0.9924	0.0062	0.0000	0.0019	0.0000	NaN
958	0.9921	0.0018	0.9780	NaN	0.9925	0.0074	0.0000	0.0019	NaN	0.0000
956	0.9920	0.0020	0.9783	NaN	0.0067	0.0000	0.0018	NaN	0.0067	0.0071
952	0.9921	0.0020	0.9781	NaN	0.9920	0.0067	NaN	0.0019	0.0000	0.0000
944	0.9925	0.0019	0.9777	NaN	0.0065	NaN	0.0018	0.0000	0.0064	0.0065
928	0.9926	0.0019	0.9775	NaN	NaN	0.0068	0.9774	0.0018	0.9927	0.9926
896	0.0000	0.0018	NaN	0.3308	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

**Table 13 Component reliability in each system state (b)**

Bar component system state	1	2	3	4	5	6	7	8	9	10
895	0.0000	0.0021	NaN	0.3318	0.0000	0.0064	0.0000	0.0019	0.0000	NaN
894	0.0000	0.0016	NaN	0.3307	0.0000	0.0069	0.0000	0.0020	NaN	0.0000
892	0.0000	0.0021	NaN	0.3304	0.0070	0.0000	0.0020	NaN	0.0065	0.0066
888	0.0000	0.0020	NaN	0.3301	0.0000	0.0070	NaN	0.0019	0.0000	0.0000
880	0.0000	0.0019	NaN	0.3304	0.0067	NaN	0.0017	0.0000	0.0064	0.0064
864	0.0000	0.0019	NaN	0.3332	NaN	0.0069	0.0000	0.0022	0.0000	0.0000
768	0.3317	NaN	0.0018	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
767	0.3309	NaN	0.0019	0.0000	0.0066	0.0066	0.0000	0.0020	0.0000	NaN
766	0.3317	NaN	0.0020	0.0000	0.0066	0.0073	0.0000	0.0017	NaN	0.0000
764	0.3313	NaN	0.0021	0.0000	0.0000	0.0000	0.0021	NaN	0.0068	0.0068
760	0.3312	NaN	0.0020	0.0000	0.0069	0.0063	NaN	0.0019	0.0000	0.0000
752	0.3299	NaN	0.0021	0.0000	0.0000	NaN	0.0020	0.0000	0.0071	0.0068
736	0.3309	NaN	0.0020	0.0000	NaN	0.0000	0.0020	0.0000	0.0068	0.0070
512	NaN	0.9777	0.0018	0.9925	0.1626	0.0000	0.0000	0.0000	0.0000	0.0000
511	NaN	0.9781	0.0018	0.9925	0.0063	0.0063	0.0000	0.0019	0.0000	NaN
510	NaN	0.9778	0.0020	0.9921	0.0067	0.0069	0.0000	0.0020	NaN	0.0000
508	NaN	0.9774	0.0020	0.9920	0.9924	0.0000	0.0021	NaN	0.0068	0.0068
504	NaN	0.9782	0.0019	0.9925	0.0069	0.0067	NaN	0.0019	0.0000	0.0000
496	NaN	0.9780	0.0018	0.9925	0.9927	NaN	0.0019	0.0000	0.0070	0.0065
480	NaN	0.9777	0.0019	0.9926	NaN	0.9928	0.0021	0.9778	0.0067	0.0063



**Figure 36 Probabilities of each system state at the end of each time period**

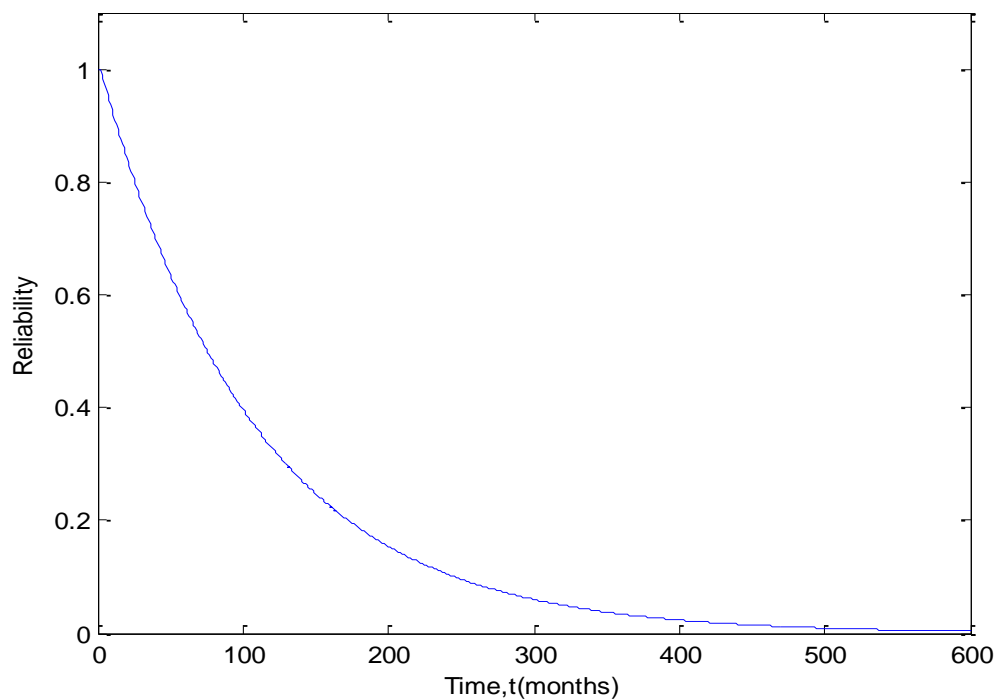
**Table 14 Most critical system states and the associated probabilities**

Time (Years)		5	10	15	20	25	30	35	40	45	50
States											
Functional states	1024	0.90134	0.51149	0.29026	0.16471	0.09347	0.05304	0.03010	0.01708	0.00969	0.00550
	960	0.00483	0.00274	0.00156	0.00088	0.00050	0.00028	0.00016	0.00009	0.00005	0.00003
sum		0.90617	0.51423	0.29181	0.16560	0.09397	0.05333	0.03026	0.01717	0.00975	0.00553
Failed states	832	0.00021	0.00113	0.00165	0.00195	0.00211	0.00221	0.00226	0.00229	0.00231	0.00232
	512	0.00370	0.00210	0.00119	0.00068	0.00038	0.00022	0.00012	0.00007	0.00004	0.00002
	448	0.00164	0.00869	0.01269	0.01496	0.01625	0.01698	0.01739	0.01763	0.01776	0.01784
	416	0.00060	0.00321	0.00469	0.00553	0.00601	0.00628	0.00643	0.00652	0.00657	0.00660
	320	0.02954	0.15858	0.23181	0.27336	0.29694	0.31033	0.31792	0.32223	0.32467	0.32606
	288	0.01975	0.10600	0.15495	0.18273	0.19849	0.20744	0.21251	0.21539	0.21703	0.21796
	192	0.03164	0.16984	0.24826	0.29277	0.31802	0.33236	0.34049	0.34510	0.34772	0.34921
160	0.00614	0.03298	0.04821	0.05685	0.06175	0.06454	0.06612	0.06701	0.06752	0.06781	
sum		0.09322	0.48253	0.70346	0.82883	0.89997	0.94034	0.96325	0.97625	0.98363	0.98782

Table 15 shows all the possible functional structural states.

**Table 15 Functional structural system states for the ten-bar truss structure**

Functional states									
1024	1023	1022	1020	1016	1008	992	991	990	988
984	976	960	959	958	956	952	944	928	896
895	894	892	888	880	864	768	767	766	764
760	752	736	512	511	510	508	504	496	480



**Figure 37 Time-variant reliability**

Figure 37 shows the time-variant system reliability over 600 months (50 years).

There are three most significant advantages of this method. The first one is its time efficiency. The simulation of reliability over 600 month finishes in only about

4.2 second after knowing the Markov matrix. The second one is that, this method can generate all the probabilities for all the system states at the end of each time period. Thirdly, the probability map reveals the time evolution of the performance of ten-bar truss structural system and provides relevant information for decision making.



### 6.3 Case study 2: analysis of a ten-bar truss structure under corrosion effects

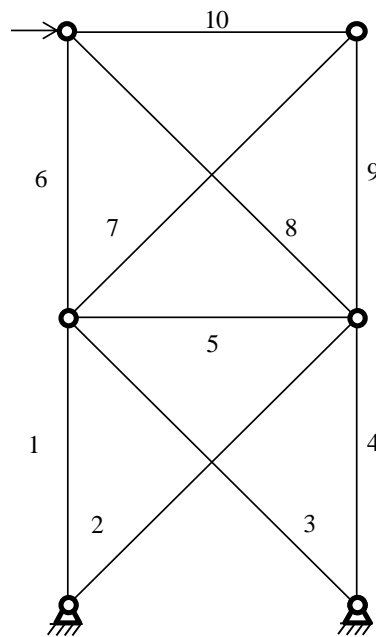


Figure 38 10-bar truss structure

Table 16 Case study 2: 10-bar truss with corrosion

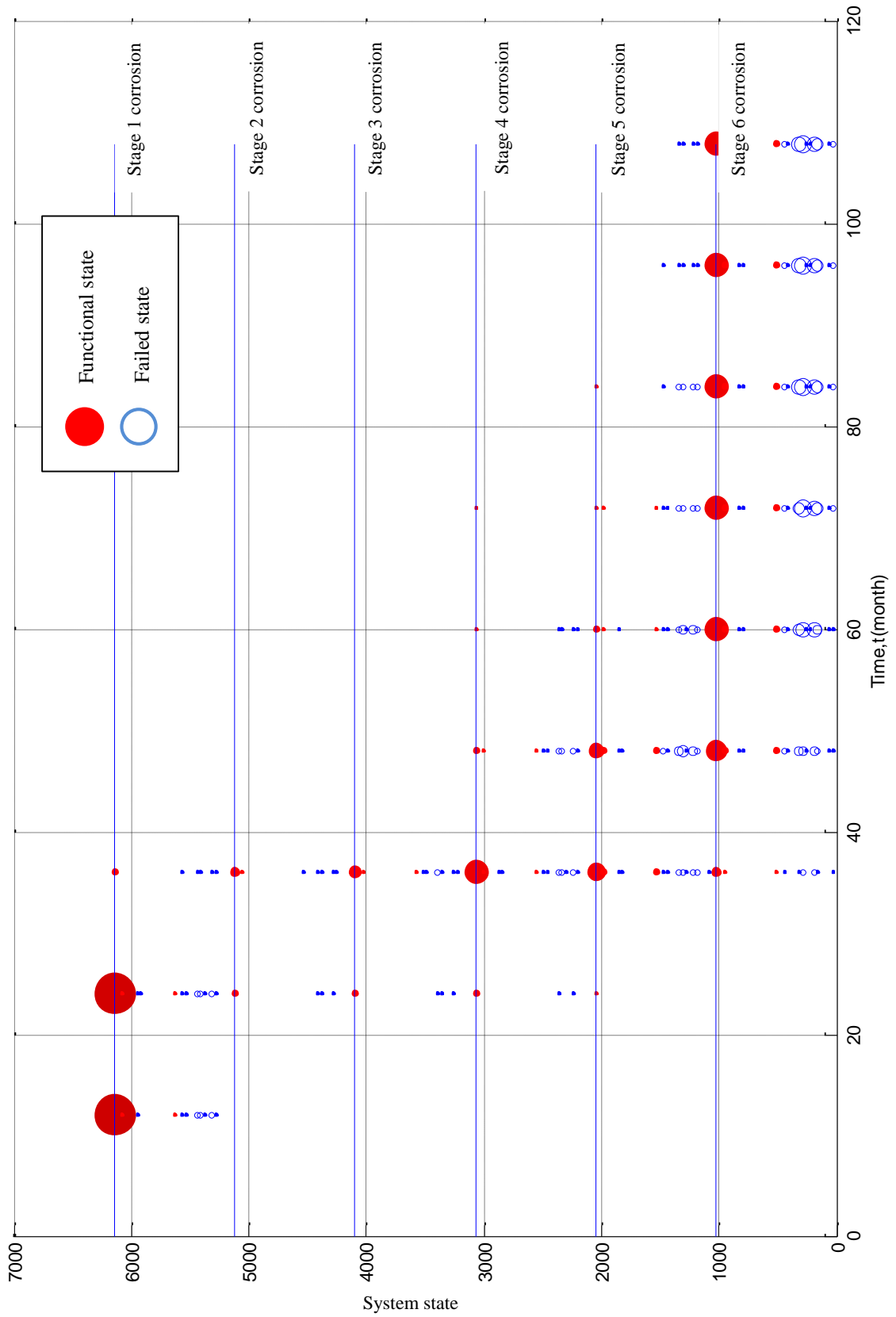
Item	Mean	COV	Type of Distribution
A1 (square meter)	4.80E-04	1.00E-01	Lognormal
A2 (square meter)	4.80E-04	1.00E-01	Lognormal
A3 (square meter)	4.80E-04	1.00E-01	Lognormal
A4 (square meter)	4.80E-04	1.00E-01	Lognormal
A5 (square meter)	3.20E-04	1.00E-01	Lognormal
A6 (square meter)	3.20E-04	1.00E-01	Lognormal
A7 (square meter)	4.80E-04	1.00E-01	Lognormal
A8 (square meter)	4.80E-04	1.00E-01	Lognormal
A9 (square meter)	3.20E-04	1.00E-01	Lognormal
A10 (square meter)	3.20E-04	1.00E-01	Lognormal
E (Pa)	2.00E+09	1.00E-02	Lognormal
Ultimate Stress (Pa)	2.35E+08	1.00E-02	Lognormal
Load (N)	5.10E+04	1.00E-01	Lognormal

According to [8], the random variables describing corrosion are listed in the table below. They are lognormally distributed.

**Table 17 Random variables describing corrosion**

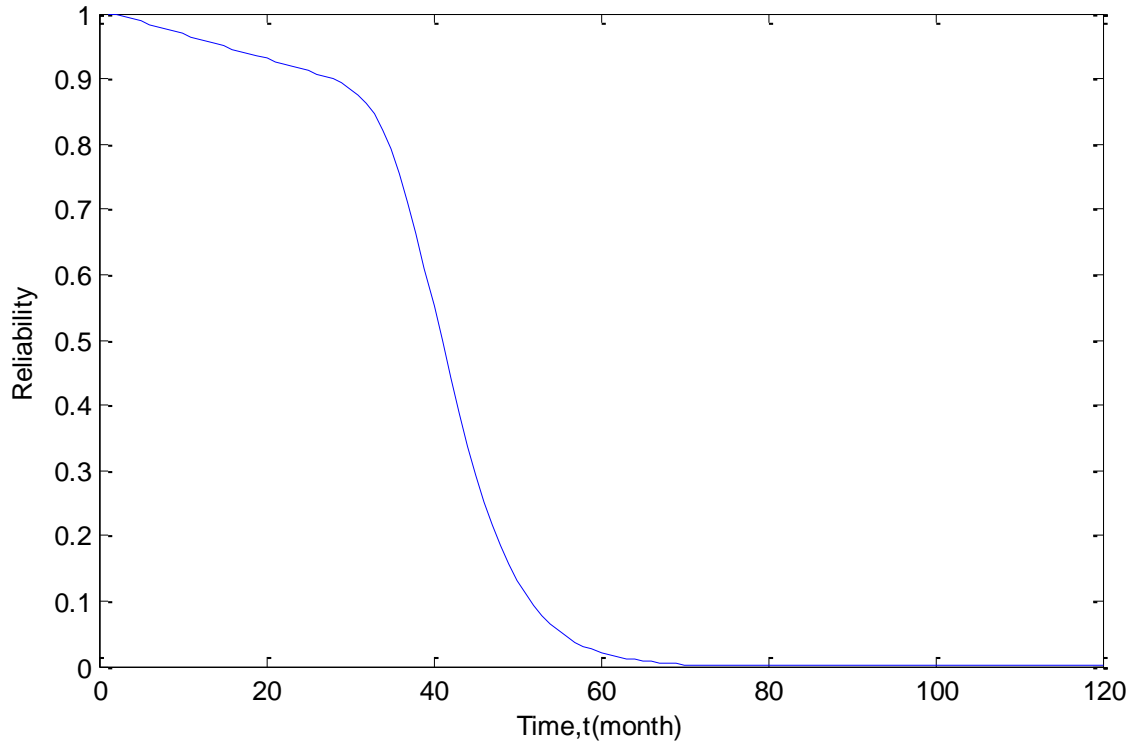
	mean	c.o.v
Corrosion rate	4.85e-7 (mm/month)	0.4
Corrosion initiation	36 (month, 3years)	0.2

The results are shown in Figure 39.



**Figure 39 Probability of each system states**

Figure 39 shows the probabilities of each system state at the end of every 12 month period. The diameter of each circle represents the corresponding probability. Corrosion condition of truss structure is divided into six categories (from stage 1 to stage 6).



**Figure 40 Time-variant structure reliability (corrosion effect considered)**

The curve in Figure 40 shows that around the 36 month, a significant drop of reliability happens. The reason for this is that the mean value of corrosion initiation time is 36 months (3 years). As soon as the corrosion started, the reliability will drop faster than before.

## Chapter 7

### Conclusions

Due to the uncertainties in real world engineering problems, it's more reasonable to design the truss structure based on its reliability. For a truss structure, its material and geometrical properties may vary with time. The long term deterioration effects such as corrosion may take place during its lifetime. For this reason, the reliability of a truss structure is also time-variant. This study has established a new way to quantify the time-variant reliability of a truss which has a large number of components. This method is appropriate for numerical computation.

Bar components are the most fundamental structural units of a truss structure. In order to evaluate the reliability of the overall truss structure, the reliabilities of each component have to be evaluated first. That is done by MCS (Monte Carlo simulation) together with FEM (finite element method). MCS provides reliable and comprehensive results of component reliability, though its speed is slow. On the other hand, the perturbation method takes less computational effort but yields a less accurate result. Thus, the MCS method is more suitable for computer simulation to quantify the component reliability for truss structures.

Markov chain is a strong mathematical tool to describe the long term behavior of a truss. As shown previously, based on the information of component reliability, the 600 month (50 years) reliability analysis can be computed in about 5 seconds (for the 10-bar truss without considering corrosion). By this method, not only the time-variant reliability has been generated, but also the probabilities of each system states have been generated as well.

There are three major advantages of this method. The first one is its time efficiency. The simulation of a deteriorating truss under corrosion over 600 months finishes in only about 5 seconds after knowing the Markov matrix. The second one is that, this method can generate all the probabilities of all the system states at the end of each time period. Thirdly, the probability map reveals the evolution of the truss structure and provides information for decision making. As mentioned in Chapter 5, it is also possible to model the maintenance of trusses by this method.

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## **Vita**

Lutong Zhang was born in Qinghai, China, the son of Juncai Zhang and YuxiaLu. After completing his work at Huangchuan High School, he went to Zhejiang University for his Bachelor of Science and finished it in 2014. Then he went to Lehigh University for the program of Master of Science.